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Research Article

The development of geometric concepts through sign language

Ezgi Özlav[1] and Mustafa Akıncı[2]

[1]*Turkish Ministry of National Education, Düzce, Türkiye (ORCID: 0009-0008-8839-9808)*
[2]*Department of Mathematics and Science Education, Zonguldak Bülent Ecevit University, Zonguldak, Türkiye (ORCID: 0000-0003-2096-7617)*

In this study on concepts of geometry, the gestures and mimics used by the mathematics teachers working in school for the deaf to define some mathematical concepts with the sign language were investigated. The research employed the structured interview technique, a qualitative research method. Data were collected through interviews and video recordings with participants. The data were analyzed using the descriptive analysis method in accordance with the parameters set by French linguist Christian Cuxac. Semiotics was utilized to conceptualize and make sense of the ideas behind the signs. As a result of the analysis, signs representing the concepts of "angle, area, diameter, circumference, and perpendicular" were suggested. It was observed that the signs shown in the Turkish Sign Language Dictionary, prepared and distributed to deaf schools by the General Directorate of Special Education and Guidance Services of the Ministry of National Education, are the same. However, the signs for the five concepts show differences. Additionally, during the interviews, teachers stated that many mathematical terms are not included in this dictionary, making it difficult for them to teach effectively. Teachers emphasized that a dictionary containing mathematical terms would be very useful in their courses.

Keywords: Emblems; Geometric concepts; Gestures; Students with hearing impairment; Signs; Sign language

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1. Introduction

Communication begins after individuals establish a relationship. Eren (1998) emphasized that to communicate, there must be an interaction between people from the outset. Communication, derived from the Latin word "communis" meaning "common", is the process of transferring information and shared understanding from one person to another (Keyton, 2005). It involves the exchange of information with other people in different forms (De Giacomo et al., 2016) and is a vital part of our daily routines (Worth, 1998). The quality of communication, which is divided into verbal and non-verbal forms, plays a significant role in determining the success of interactions between parties Zhang (2024). Aydın (2014) stated that the most important element in verbal communication is language. Verbal communication can be used by people with hearing impairments as well as individuals without hearing problems. However, it is very difficult for people with severe hearing impairments to use verbal communication. Individuals with severe hearing impairments generally prefer nonverbal communication. In nonverbal communication,

Address of Corresponding Author

Mustafa Akıncı, PhD, Ereğli Faculty of Education, Zonguldak Bülent Ecevit University, 67300, Zonguldak, Türkiye.

✉ mustafa.akinci@beun.edu.tr

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feelings and thoughts are conveyed through gestures, facial expressions, and body movements (Goldin-Meadow et al., 2012). Individuals with severe hearing impairments generally prefer lip-reading, understanding through signs, writing, and using a sign language interpreter. Gürboğa and Kargın (2003) found in their study that deaf individuals prefer sign language the most in communication.

Languages can be divided into spoken languages and sign languages. Languages such as Turkish, English, German, and French are spoken languages. Turkish Sign Language [TSL], American Sign Language [ASL], French Sign Language [LFS], and British Sign Language [BSL] are some of the sign languages. Each country has its own sign language. Sign languages also reflect the cultures of countries. Bloom and Lahey (1978) stated that language is a system with its own symbols and rules and provides information transfer between speakers of the same language through common encodings. In oral language, sounds describe concepts. For example, Turkish people call an animal that warbles and that has two legs and wings "kuş" in Turkish. In fact, this word, which consists of three sounds, has nothing to do with the animal itself. This word is a signifier belonging to it. We are the ones who give meaning to this word. This sound reminds us of the word "kuş." In English, "kuş" is symbolized as "bird," and in German as "vogel." These symbols, too, have nothing to do with warbling. It is a meaning given by people living in those countries. Sign language is also a visual language, not a verbal one, which is given to concepts using some symbols and mimics. It is a method used by the deaf to communicate. When we use the muscles on our face, our facial expressions are formed. The use of our head, feet, and hands, in other words, our body, creates gestures. The Turkish Language Association defines gesture as "an instinctive or willful movement, usually with the hand, arm, or head, to explain something." While facial expressions can only be made with facial muscles, gestures can be made with all body movements.

Arnheim (1997) stated in his study that gestures strengthen the meaning while speaking and support the statements. Krauss (1998) defined gestures as unplanned, unambiguous movements that accompany spontaneous speech and stated that they function as the visual equivalent of words in conveying verbal information. Alibali et al. (2000) emphasize that gestures are a critical component in message formation. Gestures may differ between individuals or cultures. Indeed, Kita (2009) states that there are various ways of expressing conceptual aspects of space and time in different cultures and that language structure determines how meaning is encoded through speech and gestures. Signs are gestures that are symbolic. The meanings of signs are clear and precise. They also make sense when used alone. While signs may differ between cultures, they have the same meaning among people belonging to the same culture. Lucas et al. (2003) argue that just as spoken languages vary across regions and social groups, sign languages vary depending on factors such as geography, community, and individual differences.

According to a study conducted by Karal and Çiftçi (2008), the most frequently used teaching methods by teachers working in schools for the hearing impaired are written and sign language. Ayık (2013) reported that 1.40% of teachers working in primary schools for the hearing impaired use oral methods, while 98.60% use sign-assisted methods in their lessons. Sign language is a language used by students with hearing impairment. Therefore, teachers with students with hearing impairment need to have a good command of sign language. Resorting to the finger alphabet, which is used when the meaning of a word in sign language is not known, may cause additional time loss. However, research suggests that especially iconic gestures have the ability to facilitate the understanding of concepts between individuals (Ekman & Friesen, 1969; McNeill, 1992). Familiarity with the meanings of mathematical terms in sign language will increase the efficiency and comprehensibility of mathematics lessons. Indeed, the highly iconic features of sign language offer a strong potential to capture metaphorical meaning in the learning of mathematical concepts, and this makes sign language a critical tool in the context of mathematics education (Krause & Wille, 2021). Deaf students often lag behind their hearing peers in understanding mathematical concepts and problem solving. Research emphasizes that these students

underperform throughout their academic careers and that mathematics education should be effectively supported from an early age (Leybaert & Van Cutsem, 2002; Nunes & Moreno, 2002; Titus, 1995; Traxler, 2000). Tanrıdiler (2013), in his research on mathematics teaching with students hearing impairment, stated the reasons for the difficulty of students with hearing impairment in mathematics lessons as follows: (a) lack of experience in early learning in the preschool period, (b) language deficits, (c) teachers' inability to complete the entire mathematics curriculum due to their language-oriented work, (d) the nature of mathematics language, (e) reading comprehension difficulties, and (f) difficulties in solving verbal mathematics problems. In this context, the role of sign language in mathematics learning should be re-evaluated as an important tool to overcome the barriers encountered (Krause & Wille, 2021). Deaf students have difficulty understanding abstract concepts. These students rely mostly on visual information. This leads them to learn using signs and symbols (Karal & Çiftçi, 2008). Research has shown that hearing students in mathematics classrooms use gestures as an alternative way to concretize and organize information that they cannot express verbally or formally (Alibali & Nathan, 2012; Arzarello et al., 2009). Similarly, Arzarello et al. (2005) emphasize the role of gestures in mathematical thinking, arguing that the use of signs that concretize abstract concepts makes mathematical knowledge more perceptible. While this is important even for hearing individuals, it is much more critical for individuals with hearing impairment. Since not every term in mathematics has an equivalent in sign language, teachers may need to express these terms using fingerspelling. At this point, in order to concretize abstract concepts and make them more understandable, signs corresponding to these concepts in sign language are essential for individuals with hearing impairment. In parallel, number systems and gestures in sign language are important tools for concretizing abstract mathematical concepts, which can help individuals with hearing impairment deepen their mathematical understanding (Leybaert & Van Cutsem, 2002). Gregory (1998) states that many mathematical concepts have no equivalent in sign language. The fact that mathematical terms for students with hearing impairment are not yet expressed in sign language with specific gestures increases the difficulties these students face in understanding mathematical concepts. In this context, the aim of this study is to determine how mathematical terms can be represented in sign language in the most appropriate way for students with hearing impairment to better understand mathematical concepts. This study, which was conducted with mathematics teachers working in schools in Istanbul, Bursa and Ankara, aims to develop gestures of mathematical terms that can be used in sign language and to standardize these gestures.

2. Conceptual Framework

2.1. Turkish Sign Language

Turkish Sign Language is a visual and gesture-based language used by the deaf in both the Turkish Republic of Northern Cyprus and Türkiye. Turkish Sign Language also has its own grammatical structure (Dikyuva et. al., 2015). The sign language used by each country is different: Turkish Sign Language, American Sign Language, French Sign Language, etc. Sign languages are not universal, and there are regional differences within each country (Lucas et al., 2003). The Turkish Sign Language Dictionary, prepared by the Ministry of National Education's Directorate of Special Education and Guidance Services in 2015, was created with the aim of removing regional differences in the sign language used by the hearing-impaired and standardizing sign language. There are 2,607 words in the Turkish Sign Language Dictionary.

2.2. Creation and Utilization of Sign Languages

Just as some words do not have a literal equivalent in Turkish or other languages, some words do not have a literal equivalent in TSL. In determining the sign language equivalent of a concept that is not included in TSL, some methods are used to create a signifier for the relevant concept; (a) using finger spelling, (b) making use of iconic displays, for example, representations of the words “selfie” and “laptop”, (c) using a synonym or a different word that can have similar meanings. For

example, using the word "profit" instead of "benefit" and (d) explaining the meaning of the word in a longer and indirect way, such as using "money + aid" for the word "grant".

Sign language translators use mental and physical parameters together in their translations. When translating spoken language to sign language, a transfer is made from the auditory system to a visual system, and when translating from sign language to spoken language, a transfer is made from the visual system to the auditory system. Sign language is a visual-spatial language. There are five phonological parameters that distinguish sign language from spoken language (Oral, 2015). The French linguist Christian Cuxac (1993) states these parameters as follows: (a) Hand shape: using the hand open, closed, curled, and fingers extended, curved, adjacent, closed; (b) Hand orientation: hand direction (right, left, diagonal, front, back, up, down, sideways); (c) Position: the realization area of the sign (chest, waist, head level, chin level, etc.); (d) Movement: movement of the arm, fingers, and hand, etc.; and (e) Facial expression: facial expressions, eyebrow, eye, and lip movements.

When the word "rectangle" is displayed in sign language in the dictionary prepared by the General Directorate of Special Education and Guidance Services of the Ministry of National Education, there is a picture of the word in the dictionary, the shape of the hand, and a verbal expression about how to make the sign. When this verbal expression is examined, it is seen that it has the parameters defined by the French linguist Christian Cuxac.

2.3. Semiotics: Formation and Meaning of Signs

Semiotics is a branch of science that studies how sign is formed. The American philosopher Charles Sanders Peirce initially examined the symbols and signs arising from human experiences within the framework of semiotics. Later, Swiss linguist Ferdinand de Saussure focused on linguistic signs. Signs are phenomena that represent something else. According to Saussure, signs are composed of two dimensions: the signifier and the signified. The signifier is the auditory image created by a combination of sounds, while the signified is the concept or meaning associated with it. Saussure referred to the combination of the signifier and the signified as a sign.

For example, the "tree" encompasses both dimensions. The first of these is the signifying aspect, which involves the auditory elements "t-r-e-e". This concept can also be represented by the fingerspelling alphabet or a tree gesture in sign language for individuals who are hearing and speech impaired. The second dimension is signified the concept, which is the mental image or meaning evoked by the signifier.

Semiotics, or the science of signs, encompasses a variety of meaningful units, including behaviors, languages, sign language alphabets, gestures, facial expressions, traffic signs, pictures, films, and literary works. Essentially, it covers all units that convey meaning, whether or not they are used for communication (Zorkun Çağlayan, 2017). When examining research conducted in semiotics, it is evident that much of the focus is on linguistic studies. This study, however, interprets certain mathematical terms from a semiotic perspective. The signs analyzed in this study are based on the designs and images created by the teachers.

Semiotics is the science of studying signs and how meaning is generated. It focuses on how the concept of meaning emerges rather than the specific content of the meaning itself. Semiotics examines various meaningful units, such as gestures, facial expressions, words, and music, which are used for communication. In this study, we explore gestures made by teachers in response to certain mathematical terms from a semiotic perspective.

Mathematics is fundamentally an internal symbolic activity, involving the use of written, verbal, bodily, and other signs. Semiotics aids in understanding mathematical concepts (Seeger, 2008). For instance, the expression $\frac{1}{5} \times \frac{2}{3}$ serves as a signifier representing the multiplication operation. Similarly, the expression $2 + 5$ is a text composed of three signifiers: "2", "+", "5". The semantic construction of this text represents the concept of "addition," indicating the addition process.

3. Method

3.1. Research Model

This research is a descriptive study aimed at determining the sign language representations of certain mathematical concepts used by mathematics teachers in schools for the deaf. It is also a qualitative case study because it addresses "how" and "why" questions, allowing for an in-depth examination of the phenomenon or event (Yin, 2009). Each of the ten mathematics teachers is treated as a separate case in this study, which is classified as a multiple case study design. Multiple case studies involve examining each case as a distinct and holistic entity (Merriam, 2009; Yin, 2009). This approach enables the exploration of differences both within and between cases.

Qualitative research focuses on understanding how people perceive events, describe them, and attribute meanings, rather than on statistical data analysis as seen in quantitative research (Dey, 1993). In this study, the gestures made by the teachers were investigated in depth and interpreted through a semiotic lens.

This study aims to represent and standardize some mathematical terms in sign language. To achieve this, qualitative research methods have been utilized, including structured interviews as one of the data collection methods. Interviews can be categorized in various ways; some sources classify them into forms such as interview form, conversational interview, and standardized open-ended interview (Johnson & Christensen, 2012), while others differentiate them as structured, semi-structured, and unstructured interviews (Klenke, 2008; Mitchell & Jolley, 2009).

In this study, teachers are asked about their representations of mathematical terms in sign language. To ensure comprehensive coverage and avoid overlooking any terms, a predetermined list of mathematical terms has been arranged alphabetically. Consequently, a structured interview method has been employed to facilitate consistency and thoroughness in data collection.

3.2. Participants

Purposeful sampling was used in this study to select information-rich cases for in-depth analysis, aligning with the study's objectives. Given the regional variations in sign language, mathematics teachers from schools for the deaf across different provinces were contacted. During the interviews, these teachers were asked how they represent mathematical terms in sign language in their classrooms. The mathematical terms discussed were commonly used at the middle and high school levels. Consequently, only teachers from schools specifically for the deaf were included as participants.

The interviews were conducted at the convenience of the teachers in their respective schools. Permission for the study was obtained from the Ministry of Education. A total of nine mathematics teachers from schools for the deaf participated in the study. Written consent was secured from the teachers for the use of their images. One teacher agreed to video recording under the condition that their face would not be shown, while the remaining eight teachers consented to full video recording.

3.3. Data Collection

In this research, semi-structured interviews were conducted with nine mathematics teachers working at a school for the deaf, and the interviews were recorded on video. To determine the mathematical terms included in the interview questions, four sign language dictionaries were reviewed. Based on these examinations, thirty mathematical concepts were initially identified. However, following consultations with three experts—a sign language educator, an associate professor of mathematics education, and a researcher specializing in the role of gestures in mathematics education—the list was narrowed down to twenty concepts for more effective time management. The selected mathematical concepts are: angle, area, plus, division, diameter, multiplication, circle, perimeter, subtraction, perpendicular, rectangle, minus, equal, square, square root, fraction, ratio, parallel, addition, and radius.

Before the interview, each school to be visited was contacted individually to note the available times of the teachers. Prior to starting the interviews, brief conversations were held with the teachers to inform them about the study and to establish a trust relationship by assuring them that their names would not be used in the research. Written permissions were obtained to record the interviews on video and to use their images in this and future studies. To ensure confidentiality, participants were identified with the abbreviation "Teacher" (T) followed by a number. For the study, mathematics teachers working in secondary and high schools in Ankara, Bursa, and Istanbul were asked how they expressed twenty mathematical concepts in sign language during their lessons, and their responses were recorded on video. Although data were collected on twenty mathematical concepts, this study focuses on the analysis of five selected concepts for detailed examination.

3.4. Data Analysis

In the study, gestures made by teachers for mathematical concepts were examined in both dynamic and static dimensions. The movements accompanying the formation of gestures were described using the parameters established by the French linguist Christian Cuxac. Subsequently, semiotics was employed to interpret the underlying ideas behind these movements. After identifying the gestures for the mathematical concepts from the video recordings of the nine teachers, these gestures were recorded as pictures to document the associated concept.

4. Findings

In this section, the findings on how the teachers demonstrated the 5 mathematical concepts in sign language are shared. The gestures made by the teachers in response to the concepts of angle, area, perimeter, perpendicularity, and diameter were analyzed dynamically and statically. Then, the movements that make up the gestures were described according to the parameters (hand shape, hand orientation, position, movement, and facial expression) established by Christian Cuxac. As a result of the analysis, the gestures with high frequencies of the related concepts will be presented as signs (emblems).

4.1. Findings of the Concept of Angle

It was observed that 9 teachers had 6 different signs of "angle". In Figure 8, a specific meaning could not be given to the gesture that T8 corresponded to the concept of angle. While T1 in Figure 1 has a static gesture, other teachers have performed a dynamic gesture, and this has emphasizing the dynamic definition of the angle concept. We examined the signs made by the teachers about the concept of angle.

When we examined the sign for angle of T1, it is observed that he performs a static gesture. While creating the sign, T1 likened it to the opening formed by two intersecting surfaces and creating geometrical form. For the concept of angle, the iconic indicator in Figure 1 is a sign of the angle.

Figure 1
T1's sign of angle

Right arm is straight, with the palm stretched and open; the left arm is joined at the horizontal wrists (Figure 1). A dynamic gesture was performed for the sign for angle by T2. The position of the fingers of T2 in her left hand (Figure 2a) and the brushing movement she made with the index finger of the right hand represented the space while she was making the sign for angle.

Figure 2
T2's sign of angle

(a) (b) (c) (d)

The thumb and index fingers of the left hand are extended and stretched (Figure 2a), and the right index finger touches the left index and thumb (Figure 2a, b, c, d). When the sign for angle by T3 in Figure 3 is examined, it is observed that the gap between the palms decreases and increases, indicating a dynamic gesture.

Figure 3
T3's sign of angle

(a) (b) (c) (d) (e) (f)

The Right arm upright, with the palm stretched and open, and left arm is joined at the horizontal wrists (Figure 3a). The palms close and open (Figure 3b, c, d, e, f). When we examined the sign for angle of T4, it was observed that a static gesture occurred. In Figure 4a, the position of the arms represents the right angle, in Figure 4b, the position of the arms represents the acute angle. In Figure 4c, the position of the arms explains an obtuse angle.

Figure 4
T4's sign of angle

(a) (b) (c)

The left arm is bent at the elbow at the chest level and remains parallel to the ground (Figure 4a). The right arm elbow rests on the left hand as in Figure 4a or Figure 4b or Figure 4c. Although the right and left hand usage patterns differed in the sign for angle of T5 and T2 (Figure 4), it was observed that the desired opening is the same.

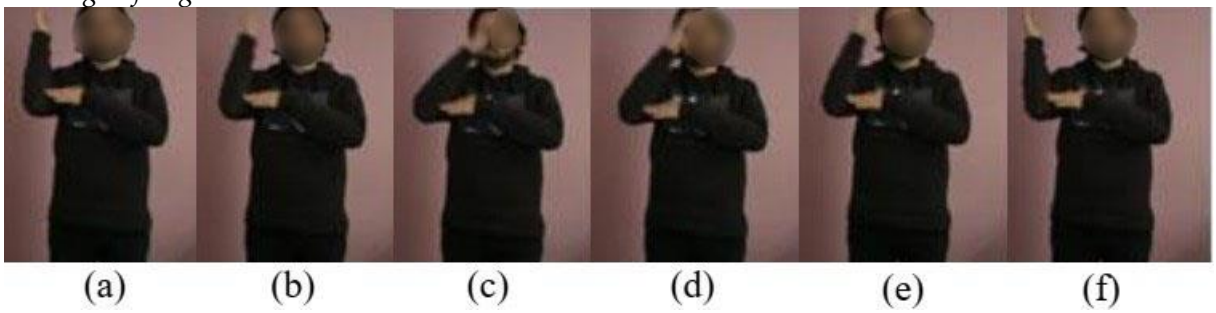
Figure 5
T5's sign of angle

The left hand's thumb and index fingers are extended and stretched (Figure 5a), and the right index finger touches left index and thumb (Figure 5b, c). It was observed that T6 drew an arc between the fingers with the left index finger in the sign for angle in Figure 6. There was dynamism and it was the left index finger that created the dynamism.

Figure 6
T6's sign of angle

The left hand is horizontal and fixed in the V position (Figure 6a). The index finger of the right hand is moved up and down between the fingers, with the index finger extended and the other fingers closed (Figure 6b, c, d). When the sign made by T7 for the angle was examined, a dynamic gesture was observed. The angle consists of two rays and here the arms represent rays. When we examine the gap between the arms, the right arm is closed and opened so that the position of the arms is a right angle in Figure 7a, a narrow angle in Figure 7c, and a wide angle in Figure 7f.

Figure 7
T7's sign of angle

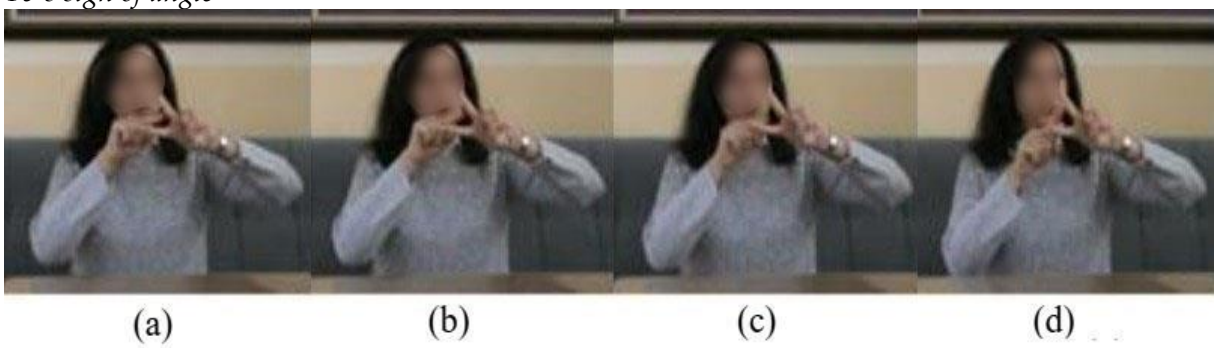


The left arm is fixed at chest level, parallel to the ground, and bent at the elbow. The right arm opens and closes on the left hand. No visual indication was found for the dynamic sign for angle made by T8.

Figure 8
T8's sign of angle

When T8's gesture for the sign for angle is examined, it is explained as "Right hand thumb and index fingers are extended and stretched, left index finger touches right index finger" depending on Cuxac's parameters. When we examine the sign for angle of T9, it is observed that they make the same sign as T6 in Figure 6. We see that it is a dynamic gesture with the movement of her index finger.

Figure 9
T9's sign of angle



The left hand is horizontal and fixed in the v position (Figure 9a). The right hand is moved up and down between the fingers, with the index finger extended and the other fingers closed (Figure 9b, c, d).

4.2. Findings of the Concept of Area

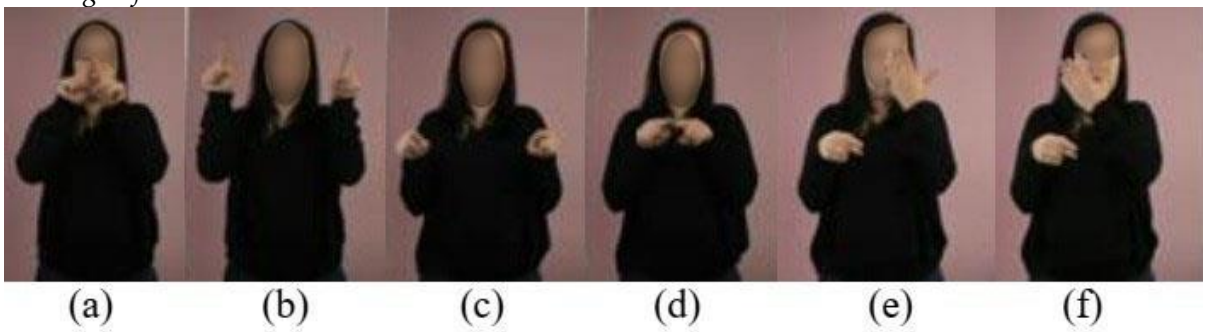
Eight out of nine teachers made signs for the concept of area. When the signs corresponding to this concept were examined, it was seen that all teachers made dynamic gestures. T1 in Figure 10, T5 in Figure 13 and T8 in Figure 16 made signs similar to the signs for angle. T3 in Figure 11, T7 in Figure 15, T9 in Figure 17, T4 in Figure 12, and T6 in Figure 14 made similar signs. We examine the visual signs made by the teachers about the concept of area.

When we examine the area-related signs of T1, it is observed that he performs a dynamic gesture. The position of the left hand represents a shallow shape or object, and the left hand brushing movement with the right hand represents the area.

Figure 10
T1's sign of area

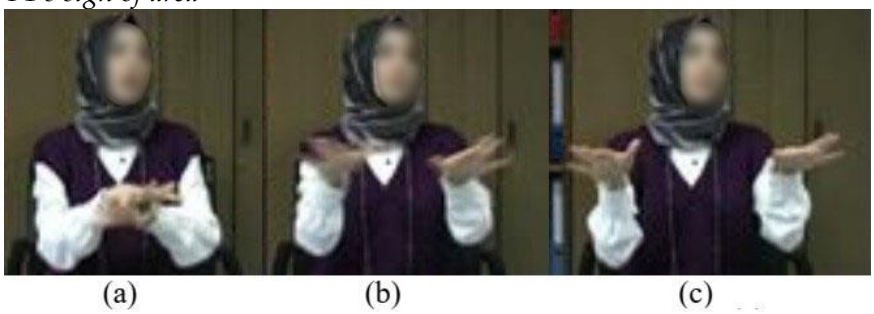
Both hands are at chest level, fingers are extended. With the left palm facing up, the right hand comes over the left hand and draws a circle on the palm (Figure 10a, b, c). When we examine the sign for area of T3, it is seen that it draws a square or rectangle-like area and then brushes it.

Figure 11
T3's sign of area



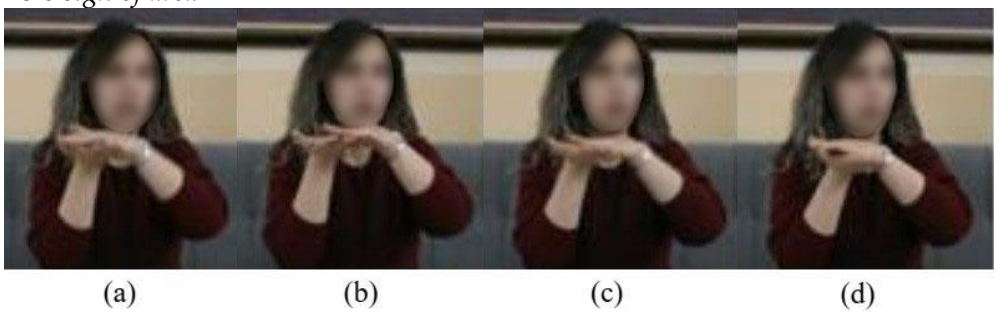
Both index fingers are moved together at chest level as in the sign, up to shoulder level (Figure 11a, b). After bringing them down, the index fingers are joined at chest level (Figure 11c, d). Then the right hand is moved from the front of the face to the right and left, with the fingers extended and the palm facing us (Figure 11e, f). In the sign for area, the right-to-left movement that T4 makes with her hands on a surface represents brushing the surface.

Figure 12
T4's sign of area



With both hands at chest level, extended, and straight and palms facing down (Figure 12a), the hands are moved right and left (Figure 12b, c). When the sign for area by T5 is examined, the position of the right hand represents a superficial shape or object, and brushing the right hand with the left hand represents the area.

Figure13
T5's sign of area

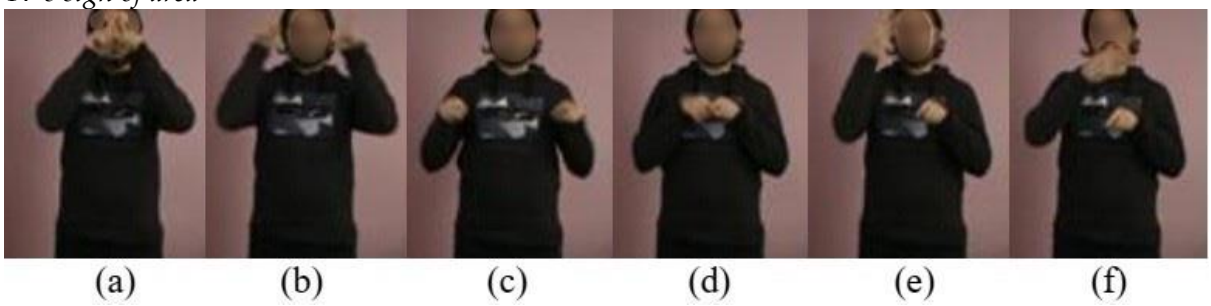


Both hands are at chest level, fingers extended and straight (Figure13a). With the right hand palm facing up, the left hand comes over the right hand and draws a circle on the palm (Figure 13b, c, d). When we examine the sign that T6 made for area, it is seen that he made the same sign as T4.

Figure 14
T6's sign of area

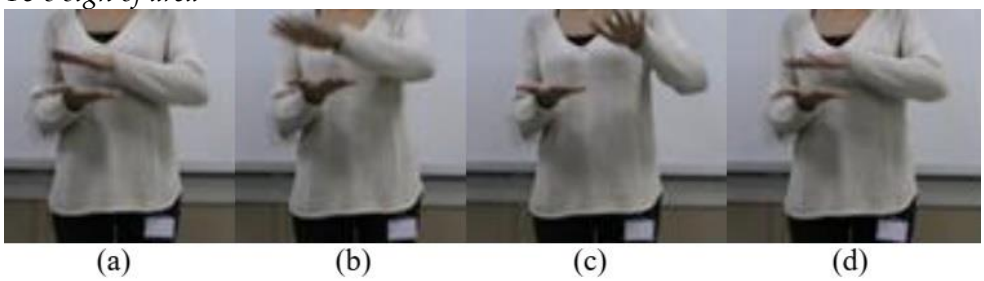
With both hands at chest level, extended straight and palms facing down (Figure 14a), the hands are spread left and right (Figure 14b, c, d, e, f). When the sign that T7 made for the angle was examined, it was determined that it was the same sign as T2. It is seen that it draws an area of a square or rectangle and then brushes this area.

Figure15
T7's sign of area



Both index fingers are moved from chest level (Figure 15a) to shoulder level as in the figure (Figure 15b). After bringing them down (Figure 15c), the index fingers are joined at chest level (Figure 15d). Then the fingers of the right hand are extended and straight, and the hand is moved to the right and left in front of the face with the palm facing us (Figure 15e, f). When we examine the sign for angle by T8, the stance of the right hand shows a surface, and the left hand drawing a circular motion on the right hand indicates that the surface has been brushed.

Figure 16
T8's sign of area



Both hands are at chest level, fingers are straight (Figure 16a). With the right hand palm facing up, the left hand draws a circle about an inch above the right hand and on the palm (Figure 16b, c, d). When the sign for angle by T9 was examined, it was seen that she brushed the square area she made with her left hand with her right hand.

The index finger of the right hand is extended, and the other fingers are closed, the little and index fingers of the left hand are extended, and the other fingers are closed. The index finger of the right hand is placed on the tips of the extended fingers of the left hand (Figure 17a). Then the right hand is moved parallel to the left hand (Figure 17b, c, d).

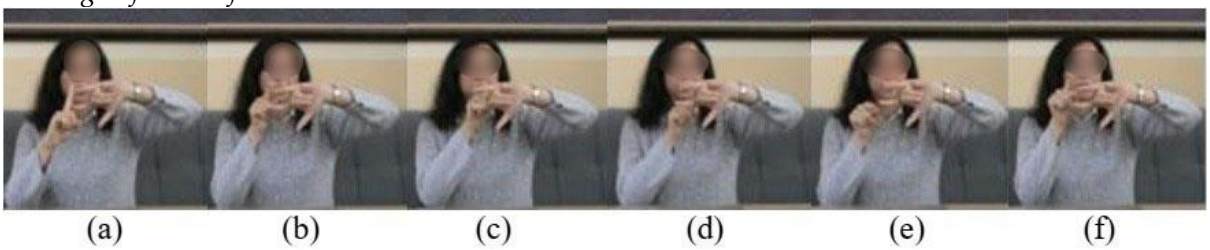
Figure 17
T9's sign of area

4.3. Findings of the Concept of Circumference

There is dynamism in the signs used by 9 teachers for the concept of circumference. Circumference tells how long we've been circling a shape. When the signs made by the teachers are examined, it is thought that they make a sign similar to this definition. We will examine the signs made by the teachers about the concept of circumference.

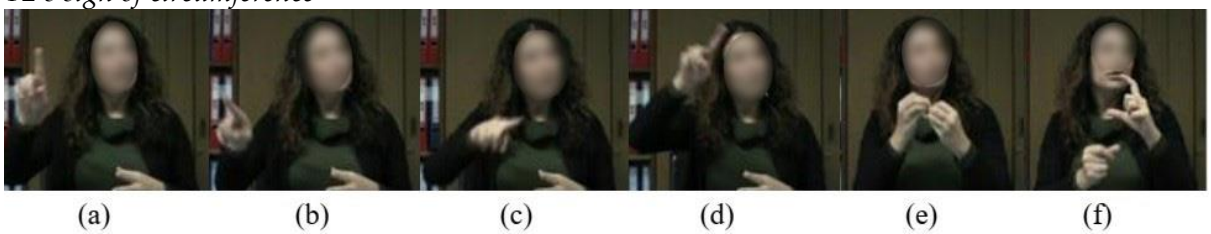
We can see in Figure 18 that T9 draws a shape and circles it with her index finger.

Figure 18
T9's sign of circumference



The index finger of the right hand is extended while the other fingers are closed. The little and index finger of the left hand are extended while the other fingers are closed. The index finger of the right hand is placed on the tips of the extended fingers of the left hand (Figure 18a). Then, the index finger of the right hand is extended and moved around the shape created (Figure18b, c, d, e, f). When we examine the sign made by T2, it is observed that she first draws a shape (Figure19a, b, c, d), then makes the sign for addition in sign language (Figure 19e), and finally forms the letter "C", which is the initial letter of the circumference (Figure 19f).

Figure 19
T2's sign of circumference

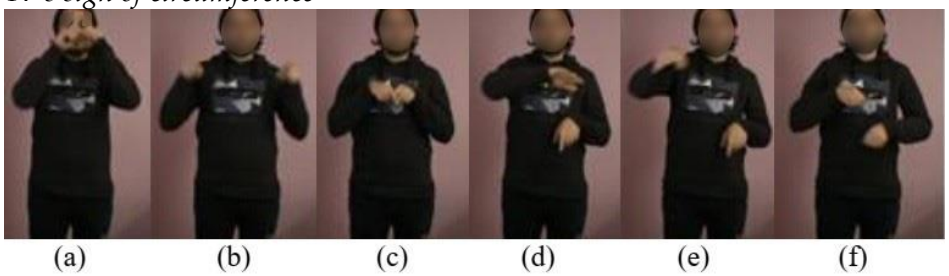


An imaginary circle is drawn with the index finger of the right hand extended and the other fingers closed (Figure 19a, b, c, d). Both hands are then open with palms facing each other, and brought close to touch each other (Figure 19e). After that, the index finger and thumb of the right hand are extended to form a "C" shape, while the other fingers are closed, and the hand is brought forward. The left hand is snapped under the right hand (Figure 19f). When examining the signs for circumference by T4 and T7, it is observed that they draw a shape and express the circumference as the sum of the perimeter of the shape they draw (Figure 20d, e, f and Figure 21d, e, f).

Figure 20
T4's sign of circumference

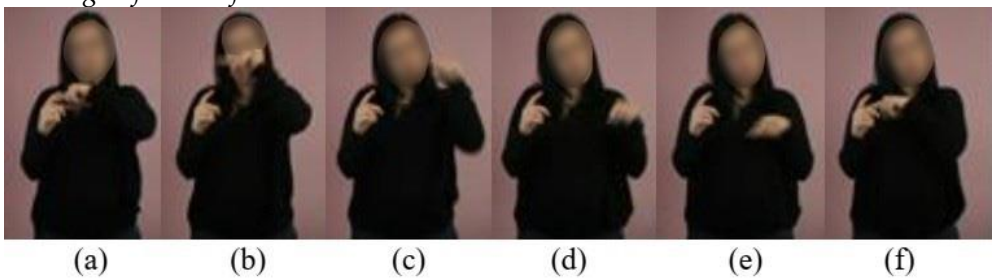
The index fingers of both hands are extended while the other fingers are closed. The hands are spread to both sides of the head, then brought down, and finally, the index fingers are joined at chest level (Figure 20a, b, c). Both hands are open and brought close together with palms facing each other (Figure 20d, e). Then, the fingertips of the right hand are together and touch the palm of the left hand (Figure 20f).

Figure 21
T7's sign of circumference



The hands are spread to both sides of the head, then pulled down, and the index fingers are joined at chest level (Figure 21a, b, c). The right hand is initially extended, turned parallel to the ground once, and then closed (Figure 21d, e, f). When examining the signs for circumference by T3 and T1, it was observed that T3 drew a shape with the index finger of the left hand, and T1 drew a shape with both the right and left index fingers. Since the circumference is the sum of the lengths of the sides of a shape, there is some deficiency in these signs regarding the concept of circumference.

Figure 22
T3's sign of circumference



An imaginary circle is drawn with the index finger of the left hand extended and the other fingers closed (Figure 22a, b, c, d, e, f).

The index fingers of both hands are extended while the other fingers are closed (Figure 23a). The hands are spread to both sides of the head (Figure 23b), then brought down (Figure 23c), and finally, the index fingers are joined at chest level (Figure 23d). When examining the signs for circumference by T5 and T8, it was observed that T5 indicated a region or circumference with the gesture made with her left hand.

Figure 23
T1's sign of circumference

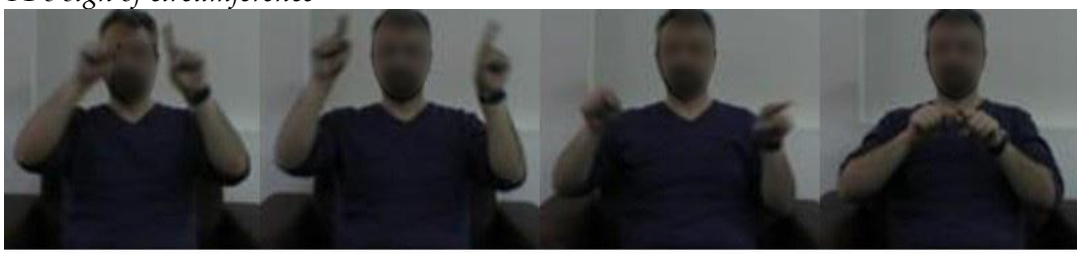


Figure 24
T5's sign of circumference

With the left hand open, straight, and palm facing down, the hand is moved to the left from chest level (Figure 24a, b, c).

Figure 25
T8's sign of circumference

With the right hand open, straight, and palm facing down, the hand is moved to the right from chest level (Figure 25a, b, c). When examining the gesture made by T6 for the concept of circumference, it is observed that he uses the circumference of his waist as an example.

Figure 26
T6's sign of circumference

The right and left index fingers are extended while the other fingers are closed. The index fingers are joined at the back in the opposite direction from the abdomen (Figure 26a, b, c).

4.4. Findings of the Concept of Perpendicular

When the signs made by the nine teachers for the concept of perpendicular are examined, the gestures made by T1, T4, T3, T7, and T8 resemble the symbol for perpendicularity. These gestures are static. The gestures made by T2, T5, and T9 are dynamic. Considering the signs made by the teachers, it appears that they use gestures in accordance with the definition of lines drawn perpendicularly to the horizontal plane. We will examine the signs made by the teachers regarding the concept of perpendicularity.

When the signs made by T1, T4, T3, T7, and T8 for the concept of perpendicular are examined, it is observed that their gestures resemble the symbol for perpendicularity. The gestures by T1 and T4 are static and are made using pointing fingers, with the postures of the two fingers likened to two perpendicular lines. In the signs made by T3 and T8, the positions of the hands and palms are similar to the perpendicularity symbol (Figure 28 and Figure 30). The position of T7's arms also represents the concept of perpendicularity (Figure 29).

Figure 27
T1's (left) and T4's (right) sign of perpendicular

The right index finger is extended while the other fingers are closed, positioned horizontally at shoulder level. The left hand is in a perpendicular position with the index finger extended and the other fingers closed. The index fingers of both hands are touched to each other as shown in the picture (Figure 27).

Figure 28
T3's sign of perpendicular

Both hands are at chest level. The left hand is straight with the palm facing down, while the right hand is straight with the palm facing left. The left hand's fingertips touch the right wrist (Figure 28).

Figure 29
T7's sign of perpendicular

Both hands are at chest level. The left hand is straight with the palm facing down, while the right hand is straight with the palm facing left. The left hand's fingertips touch the right elbow (Figure 29).

Figure 30
T8's sign of perpendicular

The left hand's fingers are extended and in a horizontal position as shown in the figure. The fingers of the right hand are also extended, and the left hand is bent at the wrist and touched to the right palm (Figure 30).

When examining the gesture that T2 made regarding the concept of perpendicularity, a dynamic process was observed. T2 brought the thumb and index finger of the left hand to the position shown in Figure 31a and created a 90-degree angle with the right index finger to demonstrate perpendicularity (Figure 31b, c, d).

Figure 31
T2's sign of perpendicular

(a) (b) (c) (d)

The thumb and index fingers of the left hand are extended, and the right index finger touches the left index finger and thumb (Figure 31). When examining the gesture of T5 for the concept of perpendicularity, the right index and thumb fingers are positioned as shown in Figure 32a. A 90-degree angle is created with the left index finger. The gesture made by T5 is a dynamic gesture.

Figure 32
T5's sign of perpendicular

(a) (b) (c) (d) (e) (f)

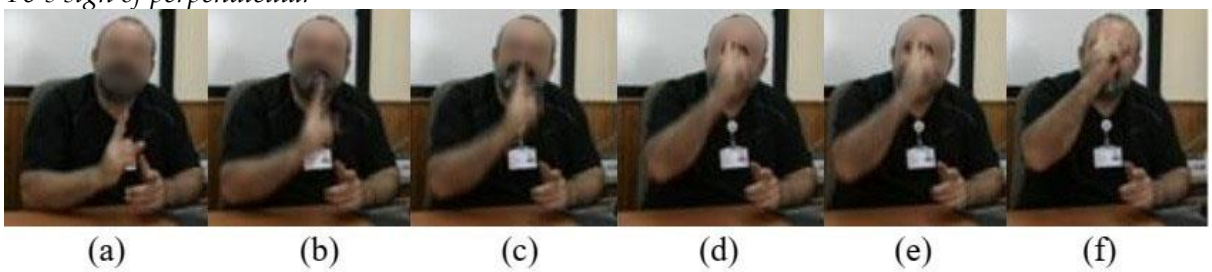
The right hand is L-shaped. With the index finger of the left hand, a sliding motion is made from the index finger of the right hand to the thumb (Figure 32a, b, c, d, e, f). The sign for T9's concept of perpendicularity emerges in a dynamic process. First, the left arm is positioned as shown in Figure 33a. The dynamic process is performed with the right hand (Figure 33b, c, d). T9 appears to be trying to dynamically describe two straight line segments that intersect each other.

Figure 33
T9's sign of perpendicular

Both hands are at chest level with fingers closed. The right hand is parallel to the left elbow (Figure 33a). The right hand is moved perpendicularly from the level of the left palm to the level of the elbow (Figure 33b, c, d), and an imaginary horizontal line is drawn to the right with the right hand (Figure 33e, f).

When examining the sign of T6's perpendicularity concept, it is observed that there is a dynamic process. T6 performs a straight line sign perpendicular to the plane from bottom to top as shown in Figure 34.

Figure 34
T6's sign of perpendicular



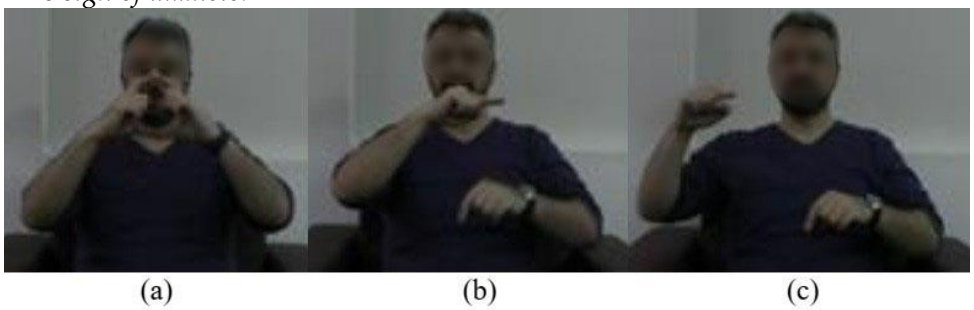
The right index finger is raised perpendicularly from the chest level to face level (Figure 34a, b, c, d, e, f).

4.5. Findings of the Concept of Diameter

Eight out of nine teachers created a gesture to signify the concept of diameter. It is thought that the teachers draw an imaginary line passing through the circle after forming the circle, in line with the definition of the diameter as "the line segment passing through the center of the circle and whose endpoints are on the circle." A dynamic gesture was observed in the signs made by seven teachers, while a static gesture was observed in T2.

In the sign that T1 made for the concept of diameter, it was seen that a circle shape was made in the first photograph. In Figure 35b and 35c, the diameter is indicated by the line dividing the circle in half.

Figure 35
T1's sign of diameter



The index fingers and thumbs of both hands are bent (forming a "C" shape), with the fingers touching each other (Figure 35a). The index finger of the right hand is moved horizontally as shown in Figure 35b and 35c. When examining the sign for diameter by T5, it is observed that she first forms a circle. Then she indicates the diameter with the line drawn over the circle using her left index finger.

Figure 36
T5's sign of diameter

The index and middle fingers of the right hand are extended and their tips touch each other (Figure 36a). The index finger of the left hand is extended while the other fingers are closed. The index finger of the left hand is then moved to the right over the fingers of the right hand (Figure 36b, c).

Figure 37
T8's sign of diameter

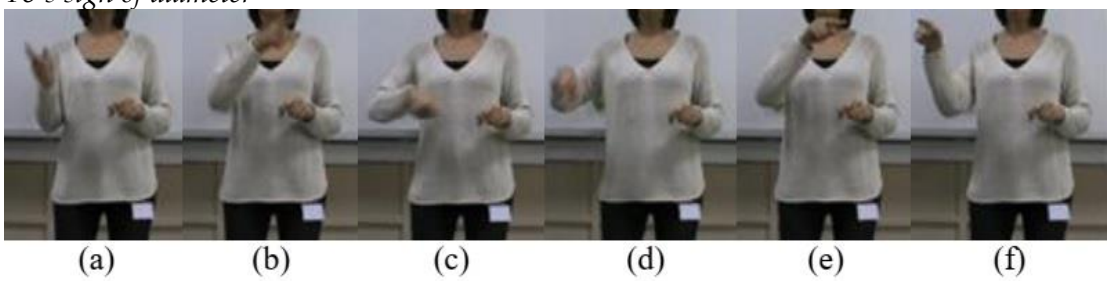
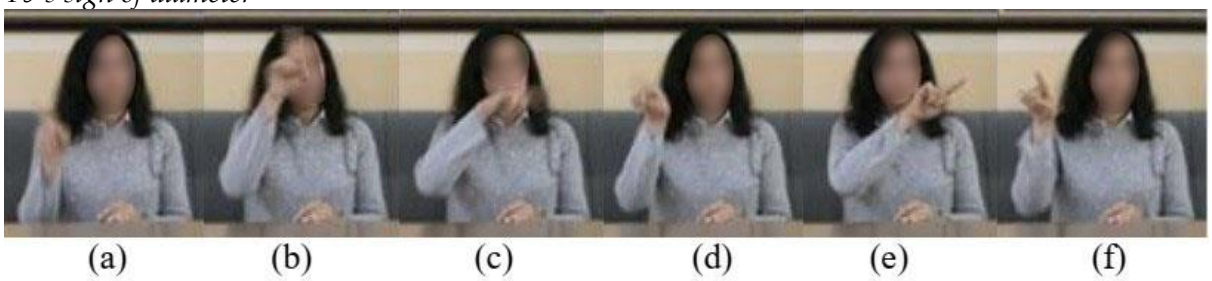


Figure 38
T9's sign of diameter



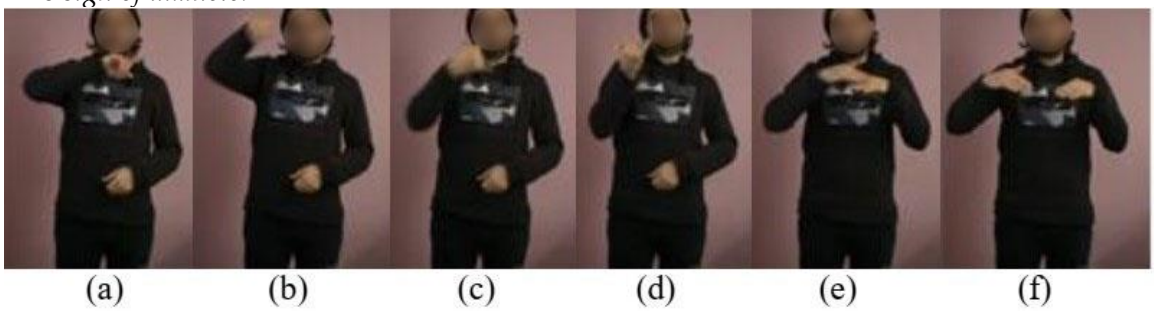
When the gestures made by T8 and T9 regarding the concept of diameter were examined, it was observed that both teachers used the same sign. After drawing a circle with the index finger, they then drew the diameter as a line passing through the middle of this circle. An imaginary circle is drawn with the index finger of the right hand extended, while the other fingers are closed (Figure 37a, b, c, d and Figure 38a, b, c, d). The index finger of the right hand is extended, and the other fingers are closed. The finger is moved from left to right at chest level (Figure 37e, f and Figure 38e, f).

When the sign for angle made by T6 was examined, it was observed that he drew a horizontal and perpendicular line with his/her index finger. This sign has not been given a specific meaning.

Figure 39
T6's sign of diameter

The index finger of the right hand is extended, and the other fingers are closed. The finger is moved from left to right at chest level (Figure 39a, b, c). The left index finger is extended, and the other fingers are closed. The left index finger is moved from top to bottom, passing the right index finger, which remains fixed, as shown in Figure 39d, e, f. When the sign for diameter made by T7 is examined, the movement he makes with his hands after drawing the circle is a sign for the diameter of the circle.

Figure 40
T7's sign of diameter



An imaginary circle is drawn with the index finger of the right hand extended, while the other fingers are closed (Figure 40a, b, c, d). With both hands at chest level, palms facing down, the right hand is moved from right to left over the left hand (Figure 40e, f). This is a static gesture that T2 uses in response to the sign for diameter. It has been observed that she made the letter "R" representing the word "diameter" in Turkish Sign Language.

Figure 41
T2's sign of diameter

The right hand is at chest level, with the index finger extended, the middle finger bent and resting on the index finger, while the other fingers are closed. Then, the index finger of the left hand is placed on the tip of the right middle finger. When the sign for diameter made by T4 was examined, it was observed that she made the letter "Ç" in Turkish Sign Language after drawing a circle with her fingers. The letter "Ç" represents the initial letter of "çap", which is the Turkish word for diameter.

Figure 42
T4's sign of diameter

The index fingers of both hands are extended, and the other fingers are closed. The hands draw a widening circle on both sides of the head (Figure 42a, b, c, d). Afterwards, the index finger and thumb of the right hand are extended, while the other fingers are closed (forming a "C" shape), and the hand is brought forward. The fingers of the left hand are then snapped under the right hand (Figure 42e, f).

5. Conclusions: Representative gestures as signs (emblems)

The gestures made by the nine teachers participating in the study related to geometric concepts are similar. These gestures can be characterized as signs (or emblems) representing the related concepts. In the study, gestures that can be characterized as signs for the concepts of angle, area, diameter, circumference, and perpendicular were identified. These gestures are shown below.

5.1. The Sign for Angle

When the signs made by the teachers in response to the concept of angle are examined, it is observed that there are four most frequently repeated signs.

Figure 43
1st sign of angle

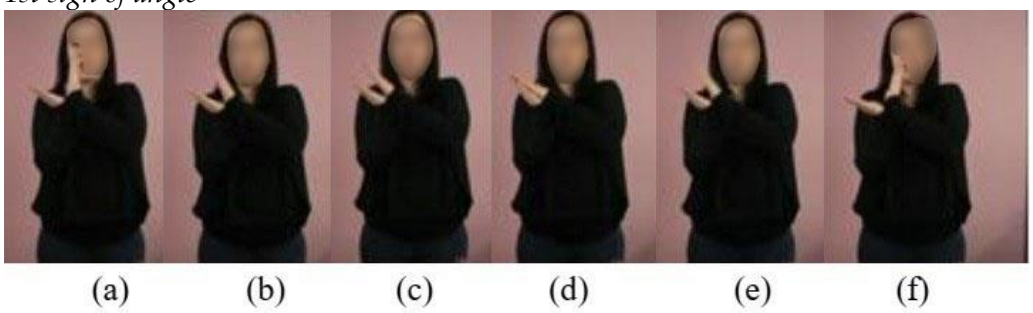
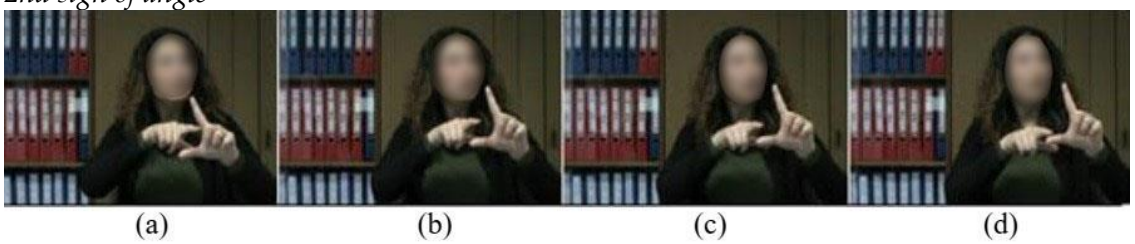


Figure 44
2nd sign of angle



The sign for angle in Figure 44 and the sign for angle in The Spread the Sign dictionary are the same.

Figure 45
3rd sign of angle

(a) (b) (c)

Figure 46
4th sign of angle

(a) (b) (c) (d)

The sign for angle in Figure 46 is the same as the sign for angle in the Texas Mathematical Sign Language dictionary.

5.2. The Sign for Area

When the gestures made by the teachers in response to the concept of area were examined, three different gestures were found to be repeated. These three gestures can be suggested as the signs for area.

Figure 47
1st sign of area

(a) (b) (c)

The notations for the concept of area in the sign language dictionaries of the Turkish Language Institution and the General Directorate of Special Education and Guidance Services are the same as the first sign for area. A flat area is depicted.

Figure 48
2nd sign of area

(a) (b) (c) (d) (e) (f)

In Figures 48e and 48f, the concept of area is described with a sign made after drawing the rectangle. The area of a surface is depicted in Figure 49.

Figure 49

3rd sign of area

(a) (b) (c) (d)

5.3. The Sign for Diameter

When the signs made by the teachers in response to the concept of diameter were examined, it was observed that T2 made the letter "R", which is the sign for the concept of diameter in sign language. After T4 drew a circle, he made the letter "Ç", which is the first letter of the word for diameter in Turkish, in sign language. It was observed that all other teachers drew an imaginary line through the circle after drawing it to represent the concept of diameter. The most frequently repeated sign for diameter is given below.

Figure 50

The sign of diameter

(a) (b) (c) (d) (e) (f)

The proposed sign for diameter is the same as the sign corresponding to the concept of diameter in the *Spread the Sign* sign language dictionary.

5.4. The Sign for Circumference

When the gestures made by the teachers in response to the concept of circumference were examined, it was observed that the most frequently repeated gesture is suggested as the sign for circumference below. In Figure 51, a geometric shape is drawn for the sign for circumference, depicted as the sum of its surroundings.

Figure 51

The sign of circumference

(a) (b) (c) (d) (e) (f)

5.5. The Sign for Perpendicular

The gestures made by the teachers in response to the concept of perpendicular were examined, and the gesture most frequently used by the teachers is suggested as the sign for perpendicular below. The sign in Figure 52 is similar to the sign corresponding to the perpendicular concept in the Spread the Sign dictionary.

Figure 52
The sign of perpendicular

6. Discussion

Mathematics teachers participating in the study used various dynamic and iconic gestures to represent mathematical concepts in sign language. Teachers used 6 gestures for the concept of angle, 4 for the concept of area, 3 for the concept of diameter, 6 for the concept of perimeter, and 4 for the concept of perpendicularity. Research on gestures in mathematics education has revealed that teacher candidates, teachers, interpreters, and students with hearing impairment use different signs (gestures) to express the same mathematical concept (Akçakoca, 2018; Akıncı & Arıkan, 2017; Glaser, 2005; Gürefe, 2015).

One of the main reasons for the difference in signs used by mathematics teachers working in schools for the hearing impaired and the students studying in these schools to denote mathematical concepts is the lack of standardized signs for mathematical concepts (Glaser, 2005; Pagliaro, 2010). Although the definitions, signs, and symbols of mathematical concepts have gained a universal character in the world literature, the fact that most of these concepts are not represented in sign language, and the ones that are, are depicted with different signs both within the same country and across the world, highlights the lack of standardization in sign language. For instance, on the *Spread the Sign* website, the same mathematical concept is represented by different signs in various countries, which clearly illustrates the absence of standardization in sign language. Similarly, the mathematics teachers participating in this study also created different gestures for the same concepts, demonstrating that this diversity exists not only at the global level but also at the national level. This variety complicates classroom communication and hinders students' with hearing impairment ability to learn mathematical concepts. The inconsistency in the signs used to teach mathematics in sign language, both nationally and globally, clearly highlights the need for standardized signs for these concepts. Indeed, Pagliaro (2010) suggests that the low levels of mathematics performance among students with hearing impairment may be linked to linguistic deficits and the challenges teachers encounter when conveying mathematical concepts in sign language. It is emphasized that the use of different signs by teachers may lead to confusion in students' understanding of mathematical concepts, thereby negatively affecting their learning process. To prevent the variety of gestural signs for mathematical concepts, a comprehensive sign language dictionary would be beneficial for both teachers working in schools for the hearing impaired and the students studying in these schools. Indeed, Gürefe (2015) emphasizes that mathematics teachers should use sign language and gestures more effectively to support students with hearing impairment in learning mathematics.

While research indicates consistency in gestural representations of some concepts learned at an early age it also shows that signs for higher-level concepts vary (Glaser 2005; Özlav 2019). It has

been observed that the gestural signs used for the mathematical concepts included in the study range from at least 3 to a maximum of 6. For example, 6 different gestural signs were proposed for the concept of angle. When these signs are examined, each can be evaluated as an iconic gesture (representation) for the concept of angle.

The variety of gestures performed by teachers for the relevant concepts may stem from their different mathematical backgrounds and cultures. Indeed, the mathematical culture and educational system individuals are exposed to determine how they teach and express mathematical concepts (Dreher et al., 2016). Chacón and Hayes (2023) noted that there is no consistency in the signs used in mathematics classes at Gallaudet University, where all mathematics courses are taught in American Sign Language, due to the different backgrounds of the instructors. This situation highlights the importance and necessity of standardizing mathematical concepts in sign language. To address this issue, a comprehensive and standardized mathematical sign language dictionary for the hearing impaired needs to be prepared. Such a dictionary would improve the educational process by enabling teachers and students to express mathematical concepts consistently.

7. Limitations and Suggestions

In this study, teachers were asked how they used 5 mathematical concepts. As the teachers did not know the sign language equivalents of some terms, they could not make a sign about the sign language equivalents of these terms. However, Karal and Çiftçi (2008) argue that the use of sign language in the education of individuals/students with hearing impairment will make the concepts easier and faster to learn and increase permanence. On the other hand, Arzarello et al. (2009) state that the use of signs in the classroom environment is a useful pedagogical resource. For this reason, teachers working in schools for the hearing-impaired should include signs of mathematical concepts in their lessons, which will create more meaningful learning for students.

The sign language dictionary distributed to hearing-impaired schools is the Turkish Sign Language dictionary prepared by the General Directorate of Special Education and Guidance Services. Many mathematical terms do not have equivalents in this dictionary. On the other hand, Gürefe (2015) and Gürboğa and Kargın (2003) found that when individuals/students with hearing impairment wanted to explain some mathematical concepts, they were forced to create a sign or gesture for that concept. For this reason, for mathematical terms that are not included in sign language dictionaries, teachers can collaborate with individuals/students with hearing impairments to develop common signs for these terms in their classes. The gestures and signs that these teachers use for new concepts can be useful for a future mathematics sign language dictionary.

This study is limited to 9 mathematics teachers working in hearing-impaired schools in Ankara, Istanbul, and Bursa. Sign language in Türkiye exhibits regional differences. A similar study can be conducted with teachers in different provinces to observe the distinct signs that may arise from these variations. In this study, we examined a sample of 9 mathematics teachers working in schools for the hearing-impaired. A parallel study can be carried out with students in schools for the hearing-impaired and can be compared with the findings of this research. Through this comparison, a more suitable sign for a concept can be determined. The study focused on 5 mathematical terms, analyzed the gestures related to these concepts, and presented the most frequently repeated gestures as signs for the respective concepts. Different studies involving other mathematical terms can also be conducted.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Declaration of interest: The authors declare that no competing interests exist.

Ethical statement: Authors declared that the study was approved by Bülent Ecevit University, Ethics Committee of Human Research on 2 June 2017 with protocol code 244.

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