Opening inquiry mathematics to parents: Can they be engaged as teachers’ partners in mathematical work?

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Abstract

This paper presents a two-stage project designed to develop the partnership between teacher and parents. The project began with a workshop constructed to motivate parents to be interested in doing mathematics in a way that is different from the one they experienced as students and, as a result, to be eager to become involved in the co-production of didactic materials for classroom use. Parents were engaged in real, collaborative, high-level mathematical work as a first step in engaging them as partners in mathematical work with their children. During this first stage, parents were familiarized with inquiry mathematics tasks to provide them with the foundation necessary to become partners and co-producers during the second. The findings give evidence that the learning of reform math tasks and their co-creation supported teacher and parents’ partnership and that parents were moved mathematically and personally by the experience.

Keywords

Parental involvement
Inquiry mathematics
Parents as partners

1. Introduction

Parental involvement in mathematics is often interpreted as simply having parents function as homework helpers. Remillard and Jackson (2006) describe a more accurate landscape that requires an alternative approach to this issue. They argue that there are two trends in education that together create a complex environment for both parents and teachers. The first relates to the necessity to engage parents in their children’s schooling in substantial ways. The second related to the fact that parents, due to reforms in mathematics education, do not feel comfortable with the mathematics their children are engaged. This emphasis on the conceptual aspect of mathematics through inquiry teaching was not apparent when the parents were in school. This leads to a dilemma: On the one hand we want parents to be engaged in their children’s learning of mathematics. On the other hand, this demands a conceptual understanding that is not possessed by parents. This makes evident that parents need opportunities to become familiar with the ideas behind the new ways of inquiry teaching. Remillard and Jackson (2006) propose the idea that an effective partnership between teachers and parents implies a collaboration among equal players. The work described in this paper accepts their thesis: conceptualizing parents as partners means taking seriously their authority with respect to their children’s learning and finding ways for parents to gain access to the discourse of inquiry teaching.

In this spirit, a two-stage project addressed to parents of elementary school students was designed to explore such an alternative approach for teacher-parent collaboration. The goal of the
first stage was to familiarize parents with tasks that their children usually face during their daily math lessons. For many parents, mathematics, although important, is often experienced as difficult, dull, boring, and based on the memorization of rules and procedures (Brown et al., 1988). Therefore, it is important first to bring them into contact with a different approach in mathematics teaching. In the second stage, parents are then invited to be co-producers who collaborate with the teacher to prepare didactic material that will be used in their children’s classroom, achieving the main object of the project of forming a partnership between the classroom teacher and the parents.

The research question was: How can the learning of inquiry math tasks and their co-creation support teacher and parents’ partnership?

The next three sections present the literature relevant to parental involvement in mathematics, the theoretical framework and the setting of the study including the tasks posed to the parents, and the methodology. Then, the findings of the study are presented, examined and commented in detail. The paper ends with some conclusions and implications for future research.

1.1. Parental involvement in mathematics: A literature review

Parental involvement in the mathematics classroom is of interest in the research community and numerous papers have presented not only a connection but a partnership among parents, teachers, and school officials. According to Epstein (1995) there are six types of involvement in a partnership program that includes parents and teachers:

- Type 1- Parenting: the basic obligation of parents to establish a supportive home environment for children,
- Type 2- Communicating: the basic obligation of schools to establish a two-way exchange (from school to home and vice versa) about school programs and children’s progress,
- Type 3- Volunteering: parents’ involvement at school, as assistants to teachers, administrators, and children in classrooms, and in areas other than school,
- Type 4- Learning at home: requiring schools to provide information and ideas to families about how to help children with their homework and other curriculum-related materials,
- Type 5- Decision making: the involvement of parents in governance and advocacy issues, taking decision making roles in advisory councils or other committees, and
- Type 6: Collaborating with the community: the integration of various community agencies and resources to support school programs.

Because Epstein’s framework evolved from many studies and from many years of work in schools, the core idea of the various programs on parental involvement (as are described below) lies within this framework. Unfortunately, ‘parental involvement’ mainly means that parents spend time with their children with their homework. Homework is the major vehicle through which parents help their children. It seems, however, that the time parents spend helping their children with mathematics homework is unrelated to children’s mathematics performance (Pedzek, Berry, & Renno, 2002), or has little effect on achievement for elementary school students (Domina, 2005).

Partnership as a component of parental involvement in mathematics classrooms must mean something more than homework support. This is why several projects invited parents to attend workshops or other programs for their own education and for being able to help their children effectively. Through their participation parents sometimes change their attitudes towards mathematics, but this is not always the case. Peressini (1998) found that many of the parents expressed anxiety about not being able to help their children (since they were not familiar with the content their children were studying) and about the apparent switch to less practice of basic computational skills. A similar result can be found in Civil’s (2001) work with parents from a working-class, largely Hispanic community. The parents’ comments described largely negative experiences in their prior learning of mathematics including, among others, lack of confidence,
feeling that they are not good at math, or feeling alienated. It seems that their difficulties are related to the conceptual understanding of mathematics since their children are engaged in activities different from those the parents had experienced as students. Bartlo and Sitomer (2008) found that the participating parents in their program tended to draw upon their own experiences with school mathematics when interpreting the mathematics that their children encounter in school (same results can be found in Quintos, Bratton, & Civil, 2005). This research indicates that parents must construct a new lens for viewing their children’s school mathematics tasks if they want to be partners in their children’s mathematics education.

However, the majority of the research studies show that the participation in these programs had a rather positive impact on parents and students. Dauber and Epstein (1990) found that through parental involvement it is possible to change students’ and parents’ attitudes toward school and mathematics. Their data clearly show that the school’s practice of informing and involving parents is more important than parent education, family size, or marital status. Shaver and Walls (1998) examined the effects of parent involvement on the reading and math achievement of low-performing students in elementary and middle grades and showed that regardless of a child’s gender or the family’s socioeconomic status, higher parent involvement increased student achievement in both reading and math. In a program conducted in Ontario, Canada (Family Math Program), evening events were organized to promote positive attitudes towards mathematics in both parents and their children, and to promote parents’ understanding of current methods of teaching mathematics (Onslow 1992). Various observations made by the researcher confirmed this change in attitude in the participants. For example, participants were less anxious as they became more comfortable experimenting with ideas and as they realized that they did not have to know the correct answer or rule immediately and that guessing and checking was a legitimate aspect of mathematics. Additionally, despite them being hesitant about attempting new activities (especially during the initial evenings), gradually they started to consistently arrive early and to confidently select various activities to play with their children. In the UK, Sangster (2004) conducted a small-scale study of the way a community primary school conveyed knowledge to parents whose children were entering their final year (10-11 year olds) about the taught mathematics of the National Numeracy Strategy. It was found that the parents learned about what their child would be doing, and gained reassurance about their own ways of calculating. They were also able to use various ways of supporting their children at home with mathematics, which is an important part of partnership. At the same time, according to the author, the children gained from working with parents in a school setting where they have autonomy. In the same spirit, using longitudinal data from elementary and secondary schools, Sheldon and Epstein (2005) examined the connections between specific family involvement activities and student achievement in mathematics at the school level. The analysis indicated that effective implementation of practices that encouraged families to support their children’s mathematical learning at home was associated with higher percentages of students who scored above proficiency on standardized mathematics achievement tests.

Parental involvement in school and positive parent-teacher interactions have also been found to positively affect teachers’ self-perceptions and job satisfaction (Hoover-Dempsey, Walker, Jones, & Reed, 2002; Tschannen-Moran & Hoy 2007). Moreover, giving parents and teachers a place to come together to learn and teach mathematics can be extremely valuable for teachers since, as Civil, Bernier and Quintos (2003) found, this gave teachers ideas that they took back into their own classrooms. Finally, another four-year project whose goal was to involve minority working class K-12 parents in supporting their children’s mathematical learning found a benefit for teachers: involving teachers in projects that have as their theoretical grounding a view of parents as intellectual resources can provide professional development for the teachers (Bernier, Allexsaht-Snider, & Civil, 2003). Parents worked on tasks very similar to what one would see in mathematics classrooms. They worked in groups, shared ideas, used manipulative materials and calculators, and were encouraged to use different representations and solution approaches. The authors state
that by giving teachers an opportunity to participate in that project, there was a greater possibility
that the teachers themselves would find and define rewards worthy of their time and effort to
support parental involvement.

Issues pertaining to parental involvement are different depending on context. This article
focuses on the context of Greece, where distinctive tensions exist regarding homework. On the one
hand, school counselors suggest that teachers avoid giving homework and have students do all
necessary work inside the classroom. On the other hand, there are teachers who, according to their
beliefs (and/or to satisfy parents’ expectations) decide to give homework. Therefore, there is a
tension about the homework landscape in Greece or about its frequency and/or its quantity.
Obviously, the issue of homework cannot be considered out of the context of its usefulness for
children’s learning. A vast array of studies has been undertaken all over the world in relation to
the factors that lead to educational achievement. In his excellent book “Visible Learning: A
synthesis of over 800 meta-analyses relating to achievement”, Hattie (2009) describes a study of
five meta-analyses that capture 161 separate studies focused on homework. According to this, the
effect size of the amount of homework at primary ages is very small compared to the

1.2. Theoretical framework

The theoretical framework for this study is the one of numeracy practices developed by Street, Baker
and Tomlin (2008). They use the term numeracy practices as “more than the behaviours that occur when
people do mathematics and more than the events in which numerical activity is involved” (p. 20). Instead,
they focus on the conceptualisations, the discourse, the values and beliefs and the social relations
that surround numeracy events, as well as the context in which they are located. A numeracy
event is a situation in which numeracy is used by participants. Numeracy practices include things
like: writing in particular ways, using language in particular ways (e.g., counting, defining, etc),
using physical objects in particular ways (e.g., counting on fingers, using a ruler or a calculator),
and using particular gestures (pointing, tapping, tracing lines, etc). These practices are different at
home and at school. Numeracy practices used at home might be more informal or involve more
heuristic methods (e.g., using approximation and estimation); those at school are more specific,
seen as ‘formal’ and often include a stronger value of right or wrong. The concept of numeracy
practices is seen as having four dimensions: (a) content, (b) context, (c) values and beliefs, and (d)
social relations. The “content” dimension refers to activities, techniques, and processes of
numeracy that individuals engage in: this is the mathematical dimension of the numeracy
practices. The “context” dimension refers to the framing of those occasions when numeracy is
studied and the purposes for that use of mathematics. Thus context is individual-dependent. The
“values and beliefs” dimension is concerned with the decisions that will be made or the concepts
that will be handled depending on the individuals’ beliefs and concepts about the nature of
numeracy; this represents the individual’s epistemology. Finally, the “social relations” dimension
involves the kinds of control over content, management of contexts and roles. It could be, for
instance (in our case), the control by teachers over parents.

The framework also makes a distinction between ‘sites’ – as the actual places where activities
take place – and ‘domains’ – as areas of activity not located in specific places. For example, school
obviously is a site where certain numeracy practices take place, but there is also the chance to
extend these practices to the home. So, in that case, we might say that “the domain of schooled
numeracy practices is to be found in the site of home” (cf p.30). An event involving purchasing objects when shopping could be seen as an instance of the domain of home numeracy practice, whereas an event in the domain of school numeracy practice is characterized as having educational purposes with a teacher in control of both social relations and knowledge. A suitable example might be the exercise of a particular skill such as “change from 10p”. This distinction does not imply that the boundaries between them are fixed. At the same time it provides us with a typology of four possibilities, i.e., school domain in a school site or in home site and home domain in a home or school site. The identification of an event, its location within one of the four cases, and its analysis in terms of the components of numeracy practices can provide a source for answering our research question.

2. Description of the study

2.1. Methodological principles

Parental involvement in this study can be seen in general under the typology of Epstein (1995). But, it is the Type 4 of Epstein’s typology that is connected to this study. According to Epstein and Sanders (2002):

“…Type 4 activities enable families to become more knowledgeable about the curriculum in specific subjects, teacher’s instructional methods,…. the work their children are doing in class…” (p.421).

However, in this study, the perspective of Type 4 is taken one step further since the parents are invited to co-produce a series of similar tasks that will be used in the classroom by the teacher to work with their children. This co-production at the same time requires making clear the theoretical assumptions concerning adults learning mathematics. Two statements proposed by Dies-Palomar, Menendez and Civil (2011) have been adopted. The first statement considers learning as the product through interaction among learners. Therefore, in working with adults we tried to emphasize and promote interactions among participants. The second statement considers learning as a social practice where everybody participates. Learning is not unidirectional from teacher to students. So, having the adults being students it was necessary to build spaces where they could feel comfortable and safe to participate and to share their own knowledge.

2.2. Participants

Twenty-four parents (9 male and 15 female) volunteered to participate in this project, which took place in a primary school in an urban area of Greece. They formed four groups of five and one group of four. All the parents were aware that their participation in this project would be a pilot for planning similar meetings in the future. All the parents had children in the same class (5th graders, 10-11 year old). The classroom teacher was the children’s full time teacher for two consecutive years and also the researcher who organized this workshop. One significant difference between the abovementioned research studies and this one is the demographics of the groups. The studies mentioned in the literature review section dealt mainly with minority or low-income populations. The situation for the current project differs in that the demographics in the specific school’s area have been the same for many years: a relatively homogeneous local population of average socio-economic and educational level, with few immigrants. Some have attained a university degree; others have finished only high school. The picture of their employment status is not clear. The majority of the women do not work either because they choose not to or they did not manage to get a job. Some of the men work in the public sector or in private companies; some are self-employed technicians, etc. Because more detailed information about the participants would add nuance to our findings, we asked for it, but did not receive it.

The teacher opened the classroom to parents in an attempt to convince them that despite the challenges of learning inquiry-based mathematics, they (and their children) could do it. This was
done, knowing that there was a risk of negatively impacting the relationship between teacher and parents if something went wrong.

2.3. Procedure

The project involved two stages. In the first, parents were asked to engage with an activity that had little to do with the ones in their children’s mathematical textbooks. The aim was to familiarize parents with an alternative approach of “doing mathematics” and to impact their attitude towards math by allowing them to engage in the subject through challenging and attractive activities. In the second part, parents were invited to be partners in the sense that they were asked to collaborate with the teacher to produce a series of activities that could be used in their children’s classroom. There were at least three obstacles that had to be taken into account (Gal & Stoudt, 1995): First, as mathematics becomes increasingly more complex, parents may not have the adequate content knowledge to help their children (Civil, 2001). Second, changes in the way mathematics is taught in schools may cause confusion or resistance by the parents (Remillard & Jackson, 2006). Third, teachers are not trained to ‘teach’ adults how to work on mathematics with their children. Parents often believe that mathematics is best learned through direct instruction – the way they learned it. Therefore, unfamiliar with the rationale behind such inquiry tasks, they are probably less able to substantially support their children’s mathematical learning in this new way. Actually, this new way of teaching mathematics uses conventions unfamiliar to the parents. Very often parents struggle to interpret the task, which looks like a new language to them. Moreover, whereas their experience as students was focused on rules and procedures, now emphasis is given to meanings and relationships. Additionally, most parents know and apply instructional approaches focused on the mastery of skills: A specific skill is taught and then a collection of tasks asks the repetition of applying the same procedure so as to ensure the mastery of the skill. These factors, also, may affect whether or not most of the parents are able to help their children learn mathematics at home.

2.4. Data collection and analysis

The participants’ discussion in each group was recorded and transcribed for the purpose of the study. It was decided to avoid video recording in order to reduce the parents’ anxiety. The teacher (who actually worked at the same time as the researcher of this study) was moving around all the time giving support whenever asked to, encouraging parents to express their own ideas to their partners, to discuss their ideas, to check the validity of their ideas, as well as, to consider together whether the proposed example would really support their didactic intention. A second researcher was invited to observe and keep records in a notebook about anything that would be considered interesting and valuable information.

The two sets of data (collected and transcribed protocols and observer’s notes) that were obtained from the second part of the workshop were then examined and coded in terms of three core ideas:

(a) Whether parents’ proposals were mathematically correct and applicable,
(b) The relation between parents’ proposals and certain parts of the official curriculum (i.e., the topics of mathematics they were supposed to address), and
(c) Identifying and locating numerical practices that offered evidence that these problems fostered parents’ engagement with mathematics.

2.5. The tasks

The two tasks that are presented in this paper come from a collection of problems posed to primary school students in Poland, Czech Republic, Netherlands, and Germany in the context of the NaDiMa Project (Natural Differentiation in Mathematics) and were also presented in a workshop by Krauthausen and Scherer (2010) that took place in the Conference of Children’s Mathematical Education (CME’10). They named the first task ‘number triangles’ but it can be met in various textbooks as arithmagon (Figure 1).
Arithmagons are equilateral triangles that are divided into three congruent kites with a number placed inside each of them. The sum of the numbers in adjacent kites is written in the boxes on the corresponding sides. The task is to find numbers to be placed in the kites so that each pair of adjacent numbers will add up to the number in the box on the side between them (Figure 1 right).

The teacher who organized the project uses this activity as an introductory one for the teaching of equations; the purpose is fivefold according to Pirie and Martin (1997): (a) to introduce students to finding numbers that fit certain rules; (b) to have them practice their general arithmetic skills, as these skills are vital in the equations approach that would be adopted; (c) to put them in situations where they can ‘see’ that the numbers they want can be got by addition or subtraction of known numbers, even though they cannot ‘see’ the actual solution number; (d) to introduce the use of a drawn square (‘box’) simply as a place that needs a number written in it; (e) to undertake activities which involved inserting numbers in expressions to check whether they had solved the arithmagons.

The task was posed during the first stage of the project. The parents initially discovered the rule that connects the numbers inside the kites and boxes and then they were asked to find a rule that would allow them to determine the numbers in the kites when given the numbers in the boxes. In the end, each group presented its solution, seeking connections between the task, their solution, and the curriculum of mathematics of their children.

The second task was named by Krauthausen and Scherer (2010) ‘Times Plus Houses’ (Figure 2).

This task was posed during the second stage of the project and parents were viewed as partners and co-producers. The first step was to discover the rule that connects the numbers in two consecutive floors of the house (i.e., the numbers in the ground floor with the numbers at the second level, and the numbers in the second level with the numbers in the roof). As can be seen, the rule is that the middle number in the ground floor is multiplied separately by the other two numbers of the floor and the product is written exactly above the two factors. Then, the number in
the roof results from adding the two numbers in the second level. After discovering the rule, parents were prompted to propose ways of using these ‘Houses’ in classroom in order to teach a new concept or to help students to develop certain mathematical skills. Numerous examples were offered in order to give parents the opportunity to be familiar with the ‘House’. Then, each group attempted to create examples showing how certain goals in mathematics teaching that were relevant to their children’s curriculum, could be achieved. Representatives of each group presented and demonstrated the examples of their groups making connections always with specific topics of mathematics.

The task was chosen because it can become the origin of a series of math activities linked to many areas of the curriculum. At the same time evidence can be gained about the practices selected by the parents in relation to ‘domains’ and ‘sites’.

3. Results

Before the official start of the workshop many of the parents confided to the teacher their lack of mathematical abilities, saying ‘they were not good at mathematics’. They trusted him since he was the teacher of their children and they had had the chance to communicate with him during the school year. So, he had to encourage them and promise that it would be a rather enjoyable experience. The classroom was organized in four groups of five and one group of four. The majority of the parents knew each other since their children were classmates.

3.1. Task 1 – Arithmagons

At the beginning most of the parents were not able to cooperate with the other members of the group. So, despite the encouragement of the teacher to communicate their ideas and to be oriented towards a commonly accepted solution path, they preferred working individually. Very soon, they discovered the rule that connected the numbers of the arithmagon in Figure 1 (left). Then, and in order to become more familiar with the task, they were asked to complete a series of different arithmagons that had some of their numbers missing (first part of this stage). Parents had to apply the rule and some necessary operations in order to find these numbers (some partial results can be found in Papadopoulos, in press) (Figure 3).

![Figure 3](image)

Figure 3. Familiarizing with arithmagons

The plan was to guide parents progressively to the general rule that can be applied in order to find the numbers inside the triangle given that the numbers in the boxes are known. Obviously it was not expected for the parents to see the task as involving linear equations (see for example, Kieren, Pirie, & Gordon Calvert, 1999). So, it was necessary to gradually guide their thinking into
an alternative approach. This is why another worksheet was given to the parents providing the numbers in the boxes and asking for the numbers in the kites. However, the numbers in the boxes were chosen deliberately so that, at least for the first examples, finding the required numbers would be easy (second part of this stage) (Figure 4). As a matter of fact, as the parents were moving from one arithmagon to another, they had to develop a strategy, step by step, that would give them the numbers in the interior of the kites.

![Figure 4](image)

**Figure 4.** Moving progressively towards the rule

During this second step a shift towards the team-work was apparent and noticed by the teacher. Parents' hesitation to expose and discuss ideas had been overcome: The proof for that was the gradual change in the level of 'noise'. Whereas there was an absolute silence in the first part, now pair conversation began taking place. Partners were working together to discover the rule that would enable them to find the terms inside the kites. This rule was not so obvious that it would be found quickly by everyone and this perhaps urged them to start sharing, explaining, defending or rejecting ideas. They continuously asked the teacher to join them in order to discuss with him queries and ideas, to validate their progress, to prevent them from being disappointed. The first three arithmagons in Figure 4 were easily solved, but this did not leave the parents with confidence in their ability to discover the rule.

Interestingly, parents asked questions about the nature of the numbers that could be used:

P.1.17 Are we allowed to use decimal numbers? Could I use for example 9.5 and 10.5 to make 20?

P.1.18 In that case why not fractions? Perhaps, we could use 19/2 and 21/2 inside the kites.

Immediately they realized that it was difficult to continue working with these numbers given that they had not yet had any evidence about the global strategy that governed the finding of the numbers in the interior of the triangle. So, some of them proposed to their team to work with trial and error.
Is it a good idea to try various numbers and see whether we can find the correct ones?

Others based their strategy on the inverse relationship between addition and subtraction: Since an addition was necessary to find the numbers in the boxes, then perhaps a subtraction was needed if one wanted to find the numbers inside the kites.

During the first example we added the numbers inside the kites in order to find the external numbers. Now, it seems to me that the situation is quite the opposite. We know the numbers that are out of the triangle and we need to find the numbers inside. So, we rather have to follow the opposite path. To subtract instead of adding...

Even though neither approach was able to lead to success, they allowed parents to reach an important intermediate finding and were crucial for discovering the final rule.

If we add the numbers inside the kites, the result will be the half of the sum of the numbers in the boxes...

This discovery was enough to drive them to start another round of attempts to find the rule. After a while, two of the teams were ready to present their version of the rule to the rest of the class. Team 1 explained their rule using the arithmagon 20-32-18 (Figure 4). Team 2 chose the arithmagon 11-10-15 in the same figure.

The three numbers in the boxes are 20, 32, and 18. Their sum is 20+32+18=70. Half of 70 is 35. This means that the three numbers inside the kites must have sum 35. We know that the numbers in kites a and b must have sum 32 (Figure 5). Thus, 35 minus 32 equals 3 and this has to be written inside the third kite. Then it is easy to find that a=17 and b=15

Something is going on with the numbers that are in one corner and in the box in the opposite side i.e., the pairs (a, 11), (b, 15) and (c, 10) (Figure 5). In our arithmagon the three numbers in the boxes are 11, 10, 15 and their sum is 36. The sum of the numbers in the interior of the triangle will be 18. Ok, 18 equals…… I have 11 and I want to get 18. How much do I need? I need 7. So, I put it here, in a. Similarly, for 15 I need 3 more and for 10 I need 8 more.

At this point, the foundation for the active participation of the parents, had been successfully established. The next step would be to demonstrate why the teacher would choose such a task to...
use in his classroom. However, this was intentionally left unanswered, and would be discussed, during the second part of the project in conjunction with the second task.

3.2. Task 2 – The Times-Plus-Houses

Now that parents had successfully completed the first part of the project, the second part was introduced. Parents were given the Times-Plus-Houses and asked to find ways to utilize them for teaching purposes.

During the first stage, parents were engaged in a collaborative solution for the arithmagon task, and they presented as a group their findings to the whole classroom (some partial results can be found in Papadopoulos, in press). They realized that their potential lack of certain mathematical abilities did not prevent them from sharing and discussing ideas with their peers in the group, and, as a result, they became more confident. They lived through the experience of being exposed to the public during their negotiations inside the group and their presentation to a large audience. This increased confidence was confirmed by the fact that when they were introduced to the Times-Plus-Houses task, they immediately started discussing and sharing with each other, and consequently shared with the teacher, a variety of interesting ideas to exploit the use of Houses in the classroom. Their collaboration generated interesting proposals; all of the teams presented their proposals to the rest of the class. Many of these proposals shared common characteristics, and at the end, they were classified into concrete teaching applications. The mathematical topics the parents decided to deal with were: (a) common factors, (b) halves and doubles, (c) analysing numbers, (d) problem solving, (e) calculating areas, and (f) distributive law of multiplication over addition: six different teaching ideas.

Common factors. The parents’ first teaching idea was connected to the practice on common factors. This proposal starts with a number inside the roof and the solver has to find the numbers that must be placed in the remaining two floors. The mathematical concept behind this idea is to analyze the number in the roof, which follows certain rules. First of all, the numbers in the middle floor must satisfy the condition that their sum must be equal to the number in the roof. At the same time, these numbers must satisfy the condition that there be a common divisor. This common divisor must be placed in the middle cell of the ground floor. Then, it is easy to find the remaining two numbers on the ground floor by the proper division. One team used a specific number as an example to explain how this can be done in the classroom (Figure 6).

![Figure 6. Explaining the common divisors example](image-url)

P.2.51 (Team 3) I put the number 100 inside the roof. So, I have to find two numbers for the middle level. I choose the numbers 60 and 40. I preferred these numbers because 20 is common factor of both. This means that 60=\(3\times20\) and \(40=2\times20\). So, I can complete the rest of the house. But, the students must be careful. They must avoid in the middle level numbers such as 39 and 61. Even
though their sum is 100 they do not have a common factor.

P.2.52 (Team 5) OK. Then you have more choices. You can keep 60 and 40 in the middle level but choose 30, 2, 20 on the ground floor.

P.2.53 (Team 5) Yes!! Or perhaps 6, 10, 4!!

We note that the team had negotiated (even though they did not name it) the concept of relatively prime numbers (i.e., two numbers that share no common positive factors except 1).

**Halves and doubles:** In the parents’ second teaching idea the focus was on the practice on halves and doubles. The teams proposed to use Houses that always had the number 2 in the middle cell in the ground floor and any number in the roof. Then, the number in the roof must be the sum of its two halves. The students have to find these two halves that must be written in the middle level. With middle numbers both even, the two side numbers on the bottom must be half of their respective middle floor numbers (Figure 7).

P.2.73 (Team 1) It is prerequisite to have the number 2 in the middle cell in the ground floor.

Now, let’s say we put the number 80 in the roof. This means I need its two halves in the first floor (i.e., two 40’s, 40+40=80) and then the half of 40 is 20 and I have to multiply it by 2 twice (i.e., 2x20=40).

![Figure 7. Explaining halves and doubles](image)

**Analysing numbers:** The third teaching idea deals with analysing numbers in many different ways, which seems to exploit the good number sense of the students, and could be considered an original idea for practicing an idea that is commonly worked on in the first grades. The house is presented already having 2 numbers in its interior: inside the roof and in the middle cell in the ground floor (Figure 8). Then, the students must complete the remaining cells.

P.2.90 (Team 4) We preferred to put the number 90 inside the roof and the number 9 in the middle cell in the ground floor. I need 10 times 9 to get 90. Consequently, the sum of the two numbers in the ground floor must be equal with 10. So, one option is to have the numbers 1 and 9 and thus, in the middle floor I must put the numbers 1 x 9 = 9 and 9 x 9 = 81. However, there are more options for the pairs in the ground floor such as the pair 2 and 8, 3 and 7, and so on.
The parents address an important skill that the educational system intends to develop from the students’ early schooling. During the first grades, one of the most common tasks is to help children find pairs of numbers with a given total. Two main instances of students practicing this skill are when the students are learning the numbers from 1 to 10 and later during the learning and practice of the times table.

*Figure 8. Analysing numbers in different ways*

**Problem solving:** The fourth teaching idea uses the Houses as a mechanism to support problem solving processes for specific kinds of problems. Here are some of the examples proposed by the parents:

- **P.2.112 (Team 5)** A word problem including numbers and facts can be given to the students and the solution could be reached through the Houses. This means that in order to solve such a problem it is necessary to apply two multiplications using both times the same number as one of the factors. The calculated products must be added and this sum will be the answer to the problem.

- **P.2.113 (continue)** Let’s give you an example. John has 3 bookshelves and Jim has 2 similar ones. In each bookshelf 20 books can be fit. Find the total number of books that can be placed on the 5 bookshelves. How much of these books belong to each child?

- **P.2.114 (continue)** However, it is possible to work on problems in another way. Here is an example: A child has 25 sweets and wants to put them into two closets. There are three shelves in the first closet and 2 shelves in the second. The number of sweets is the same for each shelf. Find the number of sweets in each closet.

In a similar spirit, the parents suggested that a series of problems can be created using a variety of objects such as sweets, toys, etc. In the first example, the ‘House’ is simply used as a vehicle for visualizing the steps of the problem solving process.

The second example is more challenging for students since the numbers that have to be chosen for the middle floor must satisfy certain criteria. This idea provides a concrete way of looking at an algebraic idea. The concrete form of this problem helps those students who are just beginning to develop the ability to problem solve using abstract ideas.

**Calculating areas:** In their fifth teaching idea, parents use the Houses for calculating the area of compound shapes. This is interesting since these shapes are not among the standard types of shapes that are taught in the classroom. Moreover, the important thing is that there are not standard formulas that could be applied for calculating their area. The proposal refers to
compound shapes that can be split into two rectangles and thus the total area is found by adding the partial areas of the rectangles. The Houses can be used when the length of one side of the first rectangle equals the length of a side of the second rectangle (Figure 9).

![Figure 9. Calculating areas of compound shapes](image)

In the example above, the compound shape was split into two rectangles with dimensions (2x5) and (10x5) respectively. By substituting these dimensions in the cells of the ground floor it is easy to calculate the area of the compound shape.

**Distributive law of multiplication over addition:** Finally, the parents suggested a teaching idea relevant to the distributive law of multiplication over addition. Some of the parents realized in a more abstract way that if the numbers in the ground floor are a, b, and c then in the middle floor the numbers that must be placed are a x b and c x b and finally, the result (a x b)+(c x b) = (a+c) x b must be written in the roof. So, they proposed to use the Houses as a way of practicing this property.

P.2.129 (Team 3) We see that the final result inside the roof can be obtained by adding the numbers in the corners in the ground floor and then by multiplying this sum by the number in the middle cell. Thus, the House can be used for practicing this property.

It is evident that they recognized the distributive law of multiplication over addition although they were not able to name it. The fact is that they did not realize that in essence all of their examples were based on the distributive law of multiplication over addition. Instead, they made a connection to arithmagons. They managed to see the application of the distributive law in the relationship between the numbers in the boxes and the numbers inside the kites. For example, they saw in arithmagon of Figure 1, that 15 = 5 x 3, 9 = 3 x 3 and 24 = 8 x 3. According to the rule 24 (= 8 x 3) = 15 + 9 = (5 x 3) + (3 x 3) = (5 + 3) x 3 = 8 x 3.

P.2.131 (Team 2) We found almost the same. More precisely, we realized that the same property can be used in the number triangles. Take a look for example at the first triangle (see Figure 1). There were 15 and 9 inside the kites and 24 in the corresponding box outside. You can see that 15 = 5 x 3, 9 = 3 x 3 and 24 = 8 x 3. Then, according to this property, it is:

24 (= 8 x 3) = 15 + 9 = (5 x 3) + (3 x 3) = (5 + 3) x 3 = 8 x 3

In summary, all these possible ways of using the Houses share a common characteristic; they are based on the distributive law of multiplication over addition. However, this does not reduce the importance of having the parents being involved in this process, which was a unique
The activities designed by the parents provide a resource that can be used to help move students developmentally from a concrete understanding of these ideas to a more abstract understanding. Besides providing the teacher with some interesting activities to use in the classroom, the parents’ excitement of discovering aspects of their children’s math lessons through the interaction with such a simple tool was itself an important achievement.

After receiving feedback from parents through private discussions in the end of the project, it became clear that whereas before entering the project they had been worried due to their lack of confidence, lack of knowledge in mathematics, and stress from the necessity of collaborating in groups, this changed totally in the end of the project. They acknowledged:

- **P.3.11** It was very important for me to work in groups because I really enjoyed supporting each other and discussing our ideas.
- **P.3.15** Feel satisfied for understanding these ‘peculiar’ tasks I often meet in my child’s notebooks.
- **P.3.20** I have to confess that I gained in confidence in relation to my own understanding of mathematical knowledge and I believe that this can be transferred now to my children at home.

The parents agreed to have informal private discussions instead of being formally interviewed, and gave their approval for these discussions to be used as data for this study.

The surprise was pleasant for the teacher too. The above-mentioned acknowledgement of the parents was considered as a positive answer to the question of whether there could be an alternative way of communicating with parents. Their lively participation and contributions to their groups was perceived as an indication that they had fun with mathematics. Obviously, this positive interaction with parents affected the self-perception and job satisfaction of the teacher who found that it was worth the risk taken. It was important for the students to see their parents collaborate with their teacher and prepare tasks for classroom use. In order to meet parents’ and students’ expectations, the teacher incorporated the parents’ proposals into the classroom teaching schedule. Immediately after completing the workshop, the teacher made connections between the proposals and the mathematical curriculum and started to plan ways to utilize this material.

4. Discussion

The group work on the tasks (and the presentation of their work to the rest of the group) can be considered numeracy events and therefore the network of the events’ dimensions should be examined. This examination will shed light on how the partnership was fostered through these events. All the teaching ideas fit the purpose of schooling and therefore, the group work constitutes an event in the domain of school numeracy practices, sited in classroom.

The content of this event is similar (but not identical) across the different teaching ideas. In the arithmagon the main concern was the distributive property of multiplication over addition, and the development of problem solving strategies. For the Times Plus Houses the content of the first teaching idea was the common factors, and the relatively prime integers. In the second teaching idea, parents focused on the practice of halves and doubles. This teaching idea allowed any number to be chosen for the roof, so it could include starting with a number on the roof, whose halves are odd numbers. In that case, it would be necessary for the students to use non-integer numbers for the ground floor. For the third teaching idea the aim is to find how a given number can be broken into two parts, which meet certain criteria (for example multiples of 9) or inversely how to combine two numbers so as to obtain another one. The next teaching idea deals with solving a certain type of problem that highlights the use of a very specific problem solving strategy based on the distributive property of multiplication over addition. The fifth idea combines the knowledge of compound shapes and the ways for calculating the area of such shapes. These ways
refer to splitting the shapes into (at least) two known sub-shapes that are simpler and familiar to the students so it could be easier for them to calculate the area of the entire shape. In our case these sub-shapes are rectangles that share a side length allowing thus the solver to apply the distributive law in order to calculate the area. Finally, in the sixth idea, parents deal with the distributive law of multiplication over addition.

The values component can be considered as ideological in the sense of what the parents see as acceptable and legitimate in mathematics. Thus, in their first teaching idea we see that even though the activity is performance-driven the parents accept the option of using tasks having more than one correct answer (P.2.51, P.2.52, P.2.53). There are numerous pairs of integers in the middle floor that could be used so as to obtain the number in roof, sharing at the same time a common factor. Moreover, these numbers satisfy the rule for the ground floor. We see the same in the analysis of 90 (second idea), again a performance driven activity with multiple solutions, and, in that sense, open-ended (P.2.90). In the fourth idea the parents see problem solving as a process that can be standardized through the use of a visual construct. The only requirement for the students is to recognize the correct cells to put the numbers involved in the problem and then to follow the rule (P.2.112). It is also interesting to notice how the parents appreciate the value of practicing on certain properties (teaching idea 6) (p. 2.129). Examining the values all ideas share, in general, their proposals were performance driven and there was a rather constant inclination to create problems that practice certain properties or skills. The fact that each idea followed a school-driven way of thinking, demonstrated an appreciation towards the work done in their children’s classroom.

In terms of social relations, the teacher began as the insider, while parents began as the outsiders. For the initial arithmagon task –the teacher began as the expert with authority, who knew the right answer, and as such had the control over the parents. He encouraged the participants, and gave immediate feedback to each group’s ideas. The groups were required to work collectively towards the discovery of the “rule” and their progress on this (no matter whether their rule was correct or not) could be considered an assessment of the possibilities for the learning of inquiry math tasks to support teacher-parents partnership.

In contrast, during the second part, the parents took control of the process even though theoretically the teacher was the expert. The teacher kept on encouraging and cajoling them, always giving positive feedback to their efforts. Gradually the teacher transformed from being the one who had to walk around trying to encourage them to an observer while the parents took on the role of guiding their own group. It became obvious that parents felt comfortable with their role of “teacher”. Especially the fifth teaching idea, with its formal character, shows that the parents have become parent-teachers, who have the authority to know the answers and to take control of the class. Also of interest is the choice of the parents to use bookshelves, sweets, and toys in their teaching ideas, since this is indicative of the well-known habit of parents to pose problems that they instantly create. This constitutes the social aspect as expressing the management of context (parents who use familiar objects from their daily experience as a context to help make problems more accessible for their children).

There is a progressive path in the social relation in this event. Initially, there was the teacher’s control over parents. The teacher was the one who was the authority and knew the right answers. And then parents started taking control, even though the teacher was still seen as the expert: Parents still needed encouragement from the teacher. Finally, as parents became more familiar with their role they started taking control over content and management which varied from resorting to familiar objects from their daily life to the use of practices that have a more formal and abstract character.

The context in this case was the school, the classroom, and, within that, the research activity. This context appeared to influence how the parents participated. All their teaching ideas tend more or less to look similar to the formal tasks included in textbooks and supposedly taught during math classrooms. The context was therefore not only physically in the school site but included the purpose of the project and the educational curriculum – important parts of the school
‘domain’. All of the teaching ideas that parents came up with were of pure arithmetical or geometrical nature that resemble ‘homework’ again demonstrating that the parents’ participation was framed by the school context. The only instance that is slightly different is the decision of parents, who created the fourth teaching idea problems, to include objects such as sweets, bookshelves and toys. These can be elements of an event found in the home ‘site’, but do not belong to the home ‘domain’ in its everyday sense (Baker, Street, & Tomlin, 2003). The purpose is still clearly focused on being a school curriculum, and therefore the context is educational rather than domestic. The parents’ way of thinking is school driven in the sense that the approaches they adopt are largely similar to the ones used in the school context. This activity suggests the possibility that numeracy practices associated with the domain of school might be found in the site of home, used by parents to help their children to learn specific mathematical concepts and skills.

All the four dimensions of these numeracy events give information on how the learning of inquiry math tasks and their co-creation can support teacher and parents’ partnership. Examining the data from the content perspective, teacher and parents dealt with a variety of mathematical topics. The opportunity given by the teacher to parents for the learning of reform math tasks made them able to co-create a multidimensional collection of types of tasks. Teacher and parents also came to share a series of beliefs concerning the value of multiple solution tasks as well as the significance of practicing on certain skills or properties. By the end of the tasks, the social relations placed much of the immediate power in the hands of the parents making them the ‘insiders’. This reflected the progressive transfer of the teacher’s control over content and management to parents. This process of parents learning to operate like teachers can be seen in the recorded interactions of the groups. Finally, the context was educational in purpose and operated as levers for fostering engagement of people who did not have a similar experience.

5. Conclusions

If new approaches in teaching mathematics are to be successful, parental involvement is vital. Numerous research studies are focused on the critical importance of parental involvement and almost all of them share three characteristics. This study takes also account of these three characteristics but the way they are approached is not the same.

First, the majority of the research studies on parental involvement in mathematics are based mainly on questionnaires and/or structured or semi-structured interviews. So, it was considered challenging to put forward an approach that would invite parents in the classroom to work together with the teacher.

Second, the studies mentioned earlier in the paper deal with groups of parents belonging to certain minorities, i.e., children in dual language (Bartlo & Sitomer, 2008), largely Hispanic community (Civil, 2001), Latino communities (Civil et al., 2003), minority working class K-12 parents (Bernier et al., 2003), parents of Mexican origin (Diez-Palomar et al., 2011), schools that served large numbers of economically disadvantaged students (Sheldon & Epstein, 2005), and African American parents (Remillard & Jackson, 2006). Perhaps it has been taken for granted that in these groups the need for involvement is more urgent. The article contributes to this research, but focuses on the need for involving parents of any classroom regardless of social or economic status.

The third characteristic is that the relationship between parents and teachers (and/or system) is focused on making parents able to support their children in homework as well as on relating students’ performance with the time parents spend on homework (Pedzak et al., 2002). In this article, the aim was to familiarize parents with both the mathematical content of their children’s mathematical education and also the methods of instruction and exploration. During the first stage the parents became familiar with inquiry mathematics tasks enabling them to gain some experience of the tasks their children face during their regular mathematics lessons. The parents’ positive response was verified through their enthusiasm and productivity during the second task in which they designed a series of teaching ideas that could be applied during the mathematical
courses of their children: practice on distributive property of multiplication over addition, on the concept of common factors, on halves and doubles, on problem solving, on calculating areas and on analysing numbers in various ways. The teaching material that was produced by parents constitutes a positive indication of another fact: parents can learn to communicate with teachers on a different level. The norm had been for parents to have regular meetings with the teacher to find out the progress or the difficulties of their children. This project, gave parents the chance to participate in a different kind of communication relevant to the approach and activities being used in their children’s classroom. The shift in parents’ attitude combined with the experience and the progressive mastery of these new tasks might help teachers to cope with parent’ resistance to their teaching methods, since parents, who have not had this sort of opportunity, very often tell their children, “I do not understand this; I was taught in a different way, and let me explain how it was”.

Thus, in order to have parents who understand mathematics as something beyond drill-and-practice, it is effective to provide opportunities for the parents to participate in a meaningful way in their children’s mathematics, by both giving them the opportunity to experience an inquiry based approach to problem solving, and soliciting their help in producing activities for their children’s classroom based on this approach. This allows parents to grasp both what their children are taught and why they are taught in that way. The results of this project show that the partnership between parents and teachers can be enriched by adding dimensions other than parents simply giving support for homework. Parents can play a vital role in providing a repository of available tasks that will help the teacher to guide their children to explore and understand mathematics. Obviously the content of the math lessons is defined by the official curricula, but the ways of approaching this content is up to the teacher or even better (as it is proposed in this paper) it is up to the successful collaboration between teachers and parents. In other words, it is up to the ability of the teachers to involve parents in a meaningful way in the co-production of ideas and tasks. Additionally, this model is not connected with the specific grade but can be equally applied to all grades for mathematics instruction.

Presumably, we cannot make generalizations since what is described here is the experience from a single attempt in a primary school in our area and the project could be better considered as a case study. However, based on these preliminary findings and on the positive feedback from parents, the school authority is planning to continue similar actions at a larger scale. The aim is to examine the new findings in order to put forward a more concrete program of partnership between school and parents. A potential problem in such plans might be how to continue supporting the enthusiasm of parents who hope to continue co-producing and using similar effective material for other mathematical problems that can be used for their children’s learning. The enthusiasm could decrease. Obviously, this paper is not able to give an answer but the problem itself can be raised.

Finally, one step further would be to study how the parents’ ideas were implemented in the class as well as the results –if there are- of how the parents’ involvement influenced their children’s learning which actually constitutes the main target of a future study.

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